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NAVAL AIR SYSTEMS COMMAND

Washington, D.C. 20360

PROFESSIONAL DEVELOPMENT CENTER

SPECIAL PROJECT NO. 69-03

THE PURE BIRTH PROCESS APPLIED TO NAVY AIRCRAFT ACCIDENTS

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JULY 1969

NAVAL AIR SYSTEMS COMMAND

THE PURE BIRTH PROCESS APPLIED TO NAVY AIRCRAFT ACCIDENTS

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NAVAIR | Navy Department

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FOREWORD

The Professional Development Center is the primary source of young civilian engineers and scientists for the Naval Air Systems Command. At one point in their training program, they undertake an original Special Project as part of the requirements for an accelerated promotion. Some of the reports on these special projects have been both interesting and informative, and deserve somewhat wider distribution. The results presented herein are not intended to reflect official US Navy policy, nor necessarily even the views of the Naval Air Systems Command. The results of the Special Project are presented herein because they are interesting, and because they may constitute a small contribution to the literature.

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ABSTRACT

Non-combat aircraft accident statistics indicate that a direct relationship exists between the number of accidents and accumulated flight hours or similarly between the accident rate and accumulated flight hours for each model of military airplane. This paper investigates the feasibility of relating accident rates directly to the total number of past accidents.

Based on the pure birth process a method for predicting aircraft accidents is presented. Application of this procedure to various test cases shows interesting and useful results. One definite conclusion that can be drawn is that with two or more years of flight and accident data, future aircraft accident rates can be predicted with fairly reliable accuracy.

Future studies based on these same procedures will delve into further relationships that may exist between aircraft characteristics and other relevant accident factors.

INTRODUCTION

United States Navy and Air Force statistics of non-combat aircraft accidents indicate that for each model aircraft some direct relationship exists between total number of accidents and accumulated flight hours or equivalently between accident rate and accumulated flight hours. Many studies have been conducted in the past to discover the nature of this relationship. The present study investigates the possibility of relating accident rates directly to total number of past accidents (instead of accumulated flight hours). The relationship between accident rate and accumulated flight hours thereby appears only as an indirect consequence of the relationship which exists between accident rate and number of past accidents.

A specific method for predicting aircraft accidents is proposed based on the pure birth process. A sample case is set-up and run to demonstrate the usefulness of the theory and the computer programs. Alternate approaches to the problem are presented for comparison and evaluation purposes. The method used for a specific case may depend on the trends demonstrated in the data.

All references in this report to aircraft accidents apply to non-combat aircraft accidents unless otherwise stated.

I. THE THEORY AND ASSUMPTIONS

The U. S. Naval Aviation Safety Center's statistics for aircraft accidents are presented by quarters for the years 1954 to 1962 and annually thereafter up to the present. For each reporting period, the number of flight hours or landings and corresponding number of accidents under various classifications (by aircraft model, damage and injury classes, fleet, etc.) are tabulated in the reports. The Safety Center's reports have been undergoing continual improvements and expanded coverage over the years so that there are certain items found in later reports that are missing in earlier reports. For purposes of this unclassified report, it shall be assumed that the flight hours T' and number N' of accidents of a specific aircraft model for a statistically significant number m' of consecutive report periods can be extracted from the Safety Center's reports and can be displayed as follows.

TABLE 1. Initial Data

Report Period Number	Flight Hours	Number of Accidents
1	T'_1	N'_1
2	T'_2	N'_2
.	.	.
.	.	.
.	.	.
m'	T'_m	N'_m

The report periods are not necessarily of the same length. This section discusses the basic statistical model assumed in this report for the analysis of such data.

The cumulative number n' of accidents and flight hours t' are defined for $K = 0,1,2,\ldots,m'$ by $n_0' = 0$, $t_0' = 0$ and for $0 < K \leq m'$,

$$n_K' = N_1' + N_2' + \ldots + N_K' ,$$
$$t_K' = T_1' + T_2' + \ldots + T_K' .$$

When n' is plotted against t', the resulting points usually appear to fall in a neighborhood of a continuous curve. (See Figure 1.) The object of this study is to find a method of defining the underlying curve so that conclusions regarding accident rates may be derived from it.

The method of analysis employed in this report requires that for each period j, $j = 1,2,3,\ldots$, the number of accidents be positive, $N_j > 0$. The data is therefore modified to eliminate any periods where $N_j' = 0$ by the following rules. If the number of accidents begins with a string of zeros, $N_1' = N_2' = \ldots = N_K' = 0$, followed by $N_{K+1}' > 0$, set

$$T_1' + T_2' + \ldots + T_K' + T_{K+1}'$$

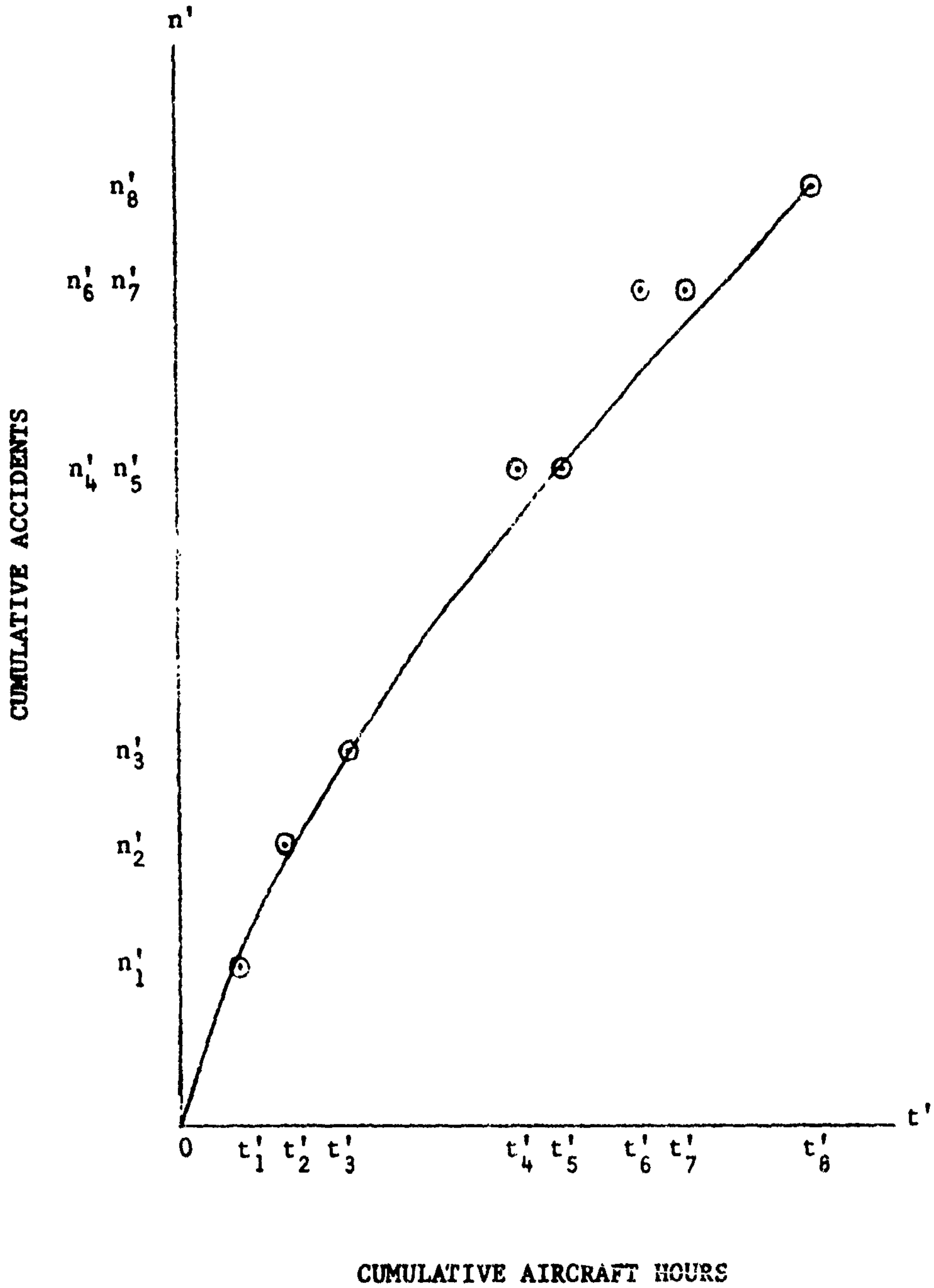
as the flight hours for a combined first period with N_{K+1}' accidents. If $N_j' > 0$ is followed by a string of zeros, $N_{j+1}' = N_{j+2}' = \ldots = N_{j+K}' = 0$ and $N_{j+K+1}' > 0$, set

$$T_{j+1}' + T_{j+2}' + \ldots + T_{j+K+1}'$$

as the flight hours for a combined period with N_{j+K+1}' accidents. If $N_j' > 0$ is followed by a string of zeros, $N_{j+1}' = N_{j+2}' = \ldots = N_{j+K}' = 0$ and N_{j+K}' is the last entry, disregard all data after period j.

FIGURE 1

INITIAL ACCUMULATED DATA GRAPH



Since the U. S. Navy statistics on accident rates are given in units of the number of accidents per 10,000 flying hours, the data must be further modified so that the flight hours are reduced to units of 10,000 hours.

TABLE 2. Modified Data

Period	Flight Hours $\times 10^{-4}$	Number of Accidents	Rate
1	T_1	N_1	N_1/T_1
2	T_2	N_2	N_2/T_2
.	.	.	.
.	.	.	.
.	.	.	.
m	T_m	N_m	N_m/T_m

Finally for $K = 0$, $n_0 = 0$ and $t_0 = 0$, and for $0 < K \leq m$,

$$n_K = N_1 + N_2 + \ldots + N_K,$$
$$t_K = T_1 + T_2 + \ldots + T_K.$$

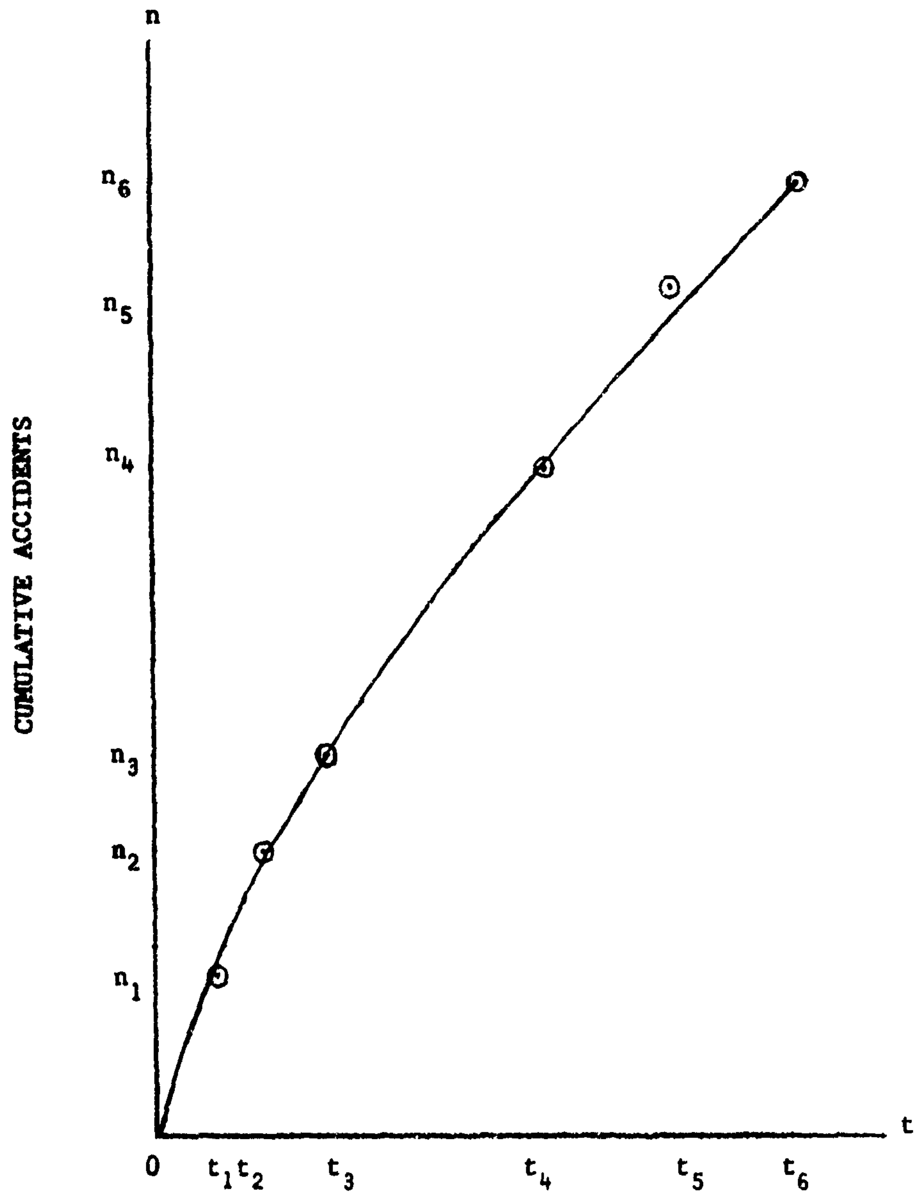
The plot of t_K versus n_K (see Figure 2) looks like the original plot of t_K' versus n_K' (Figure 1) except that points (t_j', n_j') which show no increase of n_K' with time t_j' have been eliminated and the accumulated time has been altered by the factor 10^{-4}.

Now that the problem to be investigated is set up, it is necessary to explain the theory that applies to this type of situation.

A stochastic process is an indexed family of random variables X_t on a probability space with index t ranging over a suitable parameter set T.

FIGURE 2

MODIFIED ACCUMULATED DATA GRAPH



CUMULATIVE FLIGHT HOURS X 10^{-4}

The state space of the process is a set S in which possible values of each X_t lie. In the particular case being dealt with in this report, $S = \{0,1,2,\ldots\}$ where the integers 0,1,2,... represent the accumulated number of accidents, so the process is called integer valued or a discrete state process. If $T = [0,\infty)$, as in this problem, where t is interpreted as accumulated time, then X_t is a continuous time process. A sample function of a stochastic process $\{X_t, t \in T\}$ is an assignment, to every $t \in T$, of a possible value of X_t. Given the value of X_t, such that the values of X_s, $s > t$, do not depend on values of X_u, $u < t$, then the stochastic process is Markovian. That is, a process is Markovian if the probability of any particular future behavior of the process, when its present state is known exactly, is not altered by additional knowledge concerning its past behavior:

$$\Pr\{a < X_t < b \mid X_{t_1} = x_1, X_{t_2} = x_2, \ldots, X_{t_n} = x_n\}$$

$$= \Pr\{a < X_t \leq b \mid X_{t_n} = x_n\}$$

where $t_1 < t_2 <\ldots< t_n < t$. The function

$$P(x,s;t,A) = \Pr\{X_t \in A \mid X_s = x\}, \qquad t>s$$

is called the transition probability function. A Markov process has stationary transition probability if $P(s,x;t,A)$ is a function only of $t - s$. For the special case where A is the one point set $\{j\}$,

$$P_{ij}(t) = \Pr\{X(t+u)=j \mid X(u)=i\}, \qquad i,j=0,1,2,\ldots$$

is the transition probability function for $t>0$ and is independent of $u \geq 0$.

One example of a continuous time, discrete state, Markov process is the Poisson process. If the sample function X_t counts the number of times a specified event occurs during the time period from zero to t, then each possible X_t is represented as a nondecreasing step function. The specified event occurs first at time t_1, then at time t_2, at time t_3, etc.; so the total number of occurrences of this event increases only in unit jumps, and $X_0 = 0$.

The postulates relating to the Poisson process are:

(1) The number of events happening in two disjoint intervals of time are independent. Suppose $t_0 < t_1 < t_2 <\ldots< t_n$, then increments $X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \ldots, X_{t_n} - X_{t_{n-1}}$ are mutually independent random variables.

(2) Random variable $X_{t_0+t} - X_{t_0}$ depends only on t and not on t_0 or on the value of X_{t_0}.

(3) Probability of at least one event happening in a time period of duration h is

$$\begin{aligned} p(h) &= \Pr\{X(t+h) - X(t) = 1 \mid X(t) = x\} \\ &= \lambda h + o(h), \qquad\qquad \lambda > 0. \end{aligned}$$

(4) Probability of two or more events happening in time h is o(h). This excludes the probability of the simultaneous occurrence of two or more events.

(5) $X(0) = 0$.

Using these five postulates it can be proven that X_t has a Poisson distribution with parameter λt for every t as shown by Karlin in A First Course in Stochastic Processes (pp 14-16).

Let $P_j(t)$ denote the probability that exactly j events occur in time t,

$$P_j(t) = \Pr\{X_t = j\}, \qquad j=0,1,2,\ldots.$$

Postulate (4) can be written in the form

$$\sum_{j=2}^{\infty} P_j(h) = o(h)$$

and clearly

$$p(h) = P_1(h) + P_2(h) + \ldots .$$

Due to the assumption of independence in Postulate (1)

$$\begin{aligned} P_0(t+h) &= P_0(t)\, P_0(h) \\ &= P_0(t)(1-p(h)) \end{aligned}$$

and so

$$\frac{P_0(t+h) - P_0(t)}{h} = -P_0(t)\,\frac{p(h)}{h} .$$

On the basis of Postulate (3)

$$\frac{p(h)}{h} \to \lambda .$$

Therefore probability $P_0(t)$ that the event has not happened during (0,t) satisfies the differential equation

$$P_0'(t) = -\lambda P_0(t)$$

whose solution is

$$P_0(t) = ce^{-\lambda t}.$$

The constant c is determined by the initial condition

$$P_0(0) = 1,$$

which implies that c=1. Thus

$$P_0(t) = e^{-\lambda t}.$$

Next calculate $P_j(t)$ for all j

$$P_j(t+h) = P_j(t)P_0(h) + P_{j-1}(t)P_1(h) + \sum_{i=2}^{j} P_{j-i}(t)P_i(h).$$

By definition

$$P_0(h) = 1 - p(h).$$

Postulate (4) implies

$$P_1(h) = p(h) + o(h)$$

and

$$\sum_{i=2}^{j} P_{j-i}(t)P_i(h) \leq \sum_{i=2}^{j} P_i(h) = o(h)$$

since

$$P_k(t) \leq 1.$$

By rearrangement

$$\begin{aligned} P_j(t+h)-P_j(t) &= P_j(t)[P_0(h)-1] + P_{j-1}(t)P_1(h) + \sum_{i=2}^{j} P_{j-i}(t)P_i(h) \\ &= -P_j(t)p(h) + P_{j-1}(t)P_1(h) + \sum_{i=2}^{j} P_{j-i}(t)P_i(h) \\ &= -\lambda P_j(t)h + \lambda P_{j-1}(t)h + o(h). \end{aligned}$$

Therefore

$$\frac{P_j(t+h) - P_j(t)}{h} \rightarrow -\lambda P_j(t) + \lambda P_{j-1}(t), \qquad \text{as } h \rightarrow 0,$$

resulting in

$$P_j'(t) = -\lambda P_j(t) + \lambda P_{j-1}(t), \qquad j = 1,2, \ldots$$

which is subject to the initial conditions

$$P_j(0) = 0, \qquad j = 1,2, \ldots$$

To solve this last differential equation substitute

$$Q_j(t) = P_j(t)\, e^{\lambda t}, \qquad j = 0,1,2,\ldots$$

into the differential equation $P_j'(t)$.

Then

$$Q_j'(t) = \lambda Q_{j-1}(t), \qquad j = 1,2, \ldots$$

where

$$Q_0(t) = 1$$

and the initial conditions

$$Q_j(0) = 0, \qquad j = 1,2,\ldots.$$

Solving $Q_j'(t)$ recursively

$$Q_1'(t) = \lambda \text{ or } Q_1(t) = \lambda t + c \qquad \text{so } Q_1(t) = \lambda t,$$

$$Q_2(t) = \frac{\lambda^2 \quad t^2}{2} + c \qquad \text{so } Q_2(t) = \frac{\lambda^2 \quad t^2}{2}$$

$$\vdots \qquad\qquad \vdots$$

$$Q_j(t) = \frac{\lambda^j \quad t^j}{j!}.$$

Therefore

$$P_j(t) = \frac{\lambda^j t^j}{j!} \; e^{-\lambda t} .$$

That is, for every t, X_t follows a Poisson distribution with parameter λt.

The pure birth process is a generalization of the Poisson process in which the chance of an event occurring at a given instant of time depends on the number of events that have already occurred.

A Markov process satisfying the following set of postulates is termed a pure birth process as stated by Karlin in <u>A First Course in Stochastic Processes</u>:

(1) $\Pr\{X(t+h) - X(t) = 1 \mid X(t) = k\} = \lambda_k h + o_{1,k}(h) \qquad (h \to 0+)$

(2) $\Pr\{X(t+h) - X(t) = 0 \mid X(t) = k\} = 1 - \lambda_k h + o_{2,k}(h)$

(3) $X(0) = 0$

(4) $\Pr\{X(t+h) - X(t) < 0 \mid X(t) = k\} = 0, \qquad (k \geq 0).$

Define

$$P_j(t) = P\{X(t) = j\}.$$

A system of differential equations satisfied by $P_n(t)$ for $t \geq 0$ can be derived:

$$P_0'(t) = -\lambda_0 P_0(t),$$

$$P_j'(t) = -\lambda_j P_j(t) + \lambda_{j-1} P_{j-1}(t), \qquad j \geq 1$$

with boundary conditions

$$P_0(0) = 1$$

and

$$P_j(0) = 0, \qquad\qquad j > 0.$$

If $h > 0$, $j \geq 1$ and by use of the law of total probabilities, the Markov property, and Postulate (4)

$$P_j(t+h) = \sum_{k=0}^{\infty} P_k(t) \Pr\{X(t+h) = j \mid X(t) = k\}$$

$$= \sum_{k=0}^{\infty} P_k(t) \Pr\{X(t+h) - X(t) = j-k \mid X(t) = k\}$$

$$= \sum_{k=0}^{j} P_k(t) \Pr\{X(t+h) - X(t) = j-k \mid X(t) = k\}.$$

For $k = 0,1,2,\ldots, n-2$

$$\Pr\{X(t+h) - X(t) = n-k \mid X(t) = k\}$$
$$\leq \Pr\{X(t+h) - X(t) \geq 2 \mid X(t) = k\}$$
$$= o_{1,k}(h) + o_{2,k}(h)$$

or

$$\Pr\{X(t+h) - X(t) = j-k \mid X(t) = k\} = o_{3,j,k}(h).$$

So

$$P_j(t+h) = P_j(t)\,[1-\lambda_j h + o_{2,j}(h)] + P_{j-1}(t)\,[\lambda_{j-1}h$$
$$+ o_{1,j}(h)] + \sum_{k=0}^{j-2} P_k(t)\, o_{3,j,k}(h)$$

or

$$P_j(t+h) - P_j(t) = P_j(t)[-\lambda_j h + o_{2,j}(h)] + P_{j-1}(t)[\lambda_{j-1}h$$
$$+ o_{1,j-1}(h)] + o_j(h)$$

where

$$\lim_{h\to 0} \frac{o_j(h)}{h} = 0$$

uniformly in $t \geq 0$ since $o_n(h)$ is bounded by the finite sum $\sum_{k=0}^{n-2} o_{3,j,k}(h)$ which does not depend on t.

$$\lim_{h\to 0} \frac{P_0(t+h) - P_0(t)}{h} = \lim_{h\to 0} \frac{P_0(t)[-\lambda_0 h+o_{2,0}(h)]}{h}$$

$$\lim_{h\to 0} \frac{P_j(t+h)-P_j(t)}{h} = \lim_{h\to 0} \frac{P_j(t)[-\lambda_j h+o_{2,j}(h)]+P_{j-1}(t)[\lambda_{j-1}h+o_{1,j-1}(h)]+o_j(h)}{h}$$

$$= -\lambda_j P_j(t) + \lambda_{j-1}P_{j-1}(t)$$

so

$$P_0'(t) = -\lambda_0 P_0(t)$$

and

$$P_j'(t) = -\lambda_j P_j(t) + \lambda_{j-1}P_{j-1}(t).$$

Solve this infinite set of differential equations with the initial conditions by using the integrating factor $e^{-\lambda_j t}$ for the set $\lambda_j \geq 0$

$$P_0(t) = e^{-\lambda_0 t},$$

$$P_j(t) = \lambda_{j-1}\, e^{-\lambda_j t} \int_0^t e^{\lambda_j x}\, P_{j-1}(x)\, dx, \qquad j = 1,2,\ldots\ .$$

Stating the postulates of a pure birth process in terms of aircraft accidents produces the following assumptions:

(1) Of the n_k accidents occurring up to accumulated time t_k, the last one occurred exactly at time t_k.

(2) The occurrence of accidents follows the pure birth process with rate λ_n, $n = 0,1,2, \ldots$. If $P_n(t)$ denotes the probability of exactly n accidents, $n = 0,1,2,\ldots$, by accumulated time t, then

$$\text{(i)} \quad P_0'(t) = \frac{d}{dt} P_0(t) = -\lambda_0 P_0(t)$$

$$\text{(ii)} \quad P_n'(t) = \frac{d}{dt} P_n(t) = -\lambda_n P_n(t) + \lambda_{n-1} P_{n-1}(t), \quad n \geq 1$$

where

$$P_0(0) = 1$$

and

$$P_n(0) = 0, \qquad n \geq 1.$$

The time T_k between the n_{k-1}th and n_kth accident has expected value

$$\varepsilon(T_k) = \sum_{j=n_{k-1}}^{n_k-1} \frac{1}{\lambda_j} .$$

The plot of n_k versus t_k (Figure 2) is approximately the functional relationship between the expected value of t_k ($\varepsilon(t_k)$) and n_k (regression of t_k on n_k) where

$$\begin{aligned} \varepsilon(t_k) &= \varepsilon(T_1 + T_2 + \ldots + T_k) \\ &= \varepsilon(T_1) + \varepsilon(T_2) + \ldots + \varepsilon(T_k) \\ &= \sum_{j=0}^{n_1-1} \frac{1}{\lambda_j} + \sum_{j=n_1}^{n_2-1} \frac{1}{\lambda_j} + \ldots + \sum_{j=n_{k-1}}^{n_k-1} \frac{1}{\lambda_j} \\ &= \sum_{j=0}^{n_k-1} \frac{1}{\lambda_j} . \end{aligned}$$

After examination of several plots of n_k versus t_k ($\approx \varepsilon(t_k)$) for aircraft accident data, the functional relationship

$$\lambda_n = \alpha + \gamma\mu^n, \qquad n=0,1,2,\ldots$$

was chosen to express the relationship between n_k and λ_{n_k}.

If $\alpha \geq 0$, $\gamma \geq 0$, and $0 < \mu < 1$, then α is the limiting rate; $\alpha + \gamma$ is the initial rate; and μ^n is the fractional part of γ remaining after the nth accident.

If $\mu = 1$, then

$$\begin{aligned} \lambda_n &= \alpha + \gamma \\ &= \lambda, \end{aligned}$$

λ being a constant. This is the case where the number of accidents up to accumulated flight time t has the Poisson distribution with parameter λt, and the accumulated flight time up to accident number n has the gamma distribution.

If $\gamma > 0$ and $\mu > 1$, the number of accidents as a function of t will blow-up, i.e. there will be a positive probability that the number of accidents will become infinite in finite time.

If $\alpha = 0$, $\gamma > 0$, and $\mu < 1$, the rate

$$\lambda_n = \gamma\mu^n$$

will approach zero as a limit, but the number of accidents as a function of t will be unbounded (logarithmic increase).

II. ESTIMATION OF THE ACCIDENT RATE PARAMETERS

For data such as that indicated in Table 2, when the accident rate between the nth and n + 1st accidents is taken to be of the form

$$\lambda_n = \alpha + \gamma\mu^n, \qquad n = 0,1,2,\ldots,$$

the computations described in this section will result in a least squares estimation of the unknown parameters α, γ, and μ. The choice of a quantity $Q = Q(\alpha,\gamma,\mu)$ to be reduced to a minimum by the solution α, γ, and μ is motivated by the formula

$$\varepsilon(T_k) = \sum_{j=n_{k-1}}^{n_k-1} \frac{1}{\lambda_j}$$

discussed in section I, and a desire for a close fitting cumulative number of accidents versus cumulative flight hours curve to the data points of Figure 2.

The least square estimates of α, γ, and μ will be taken to be those which minimize the quantity

$$Q = \tfrac{1}{2} \sum_{k=1}^{m} \{ \sum_{j=0}^{n_k-1} \frac{1}{\lambda_j} - t_k \}^2$$

$$= \tfrac{1}{2} \sum_{k=1}^{m} E_k^{\,2}.$$

The minimizing α, γ, and μ are to be found by refining some initial estimates α_0, γ_0, and μ_0 by Newton-Raphson iterations. Initial estimates may be obtained by examining a n_k versus observed accident rate N_k/T_k

(see Table 2) scatter diagram. With approximate smooth values of the accident rate for $n_k = 0$, n_k = some intermediate value, and $n_k = \infty$ (assuming $\mu < 1$), solve for α_0, γ_0, and μ_0.

The minimizing α, γ, and μ for Q satisfy the equations

$$\frac{\partial Q}{\partial \alpha} = \frac{\partial Q}{\partial \gamma} = \frac{\partial Q}{\partial \mu} = 0.$$

On the other hand, for points (α, γ, μ) generally, in a neighborhood of (α_0, γ_0, μ_0), the Taylor expansion of $\frac{\partial Q}{\partial \alpha}$ about the point (α_0, γ_0, μ_0) is given by

$$\frac{\partial Q}{\partial \alpha} = \left(\frac{\partial Q}{\partial \alpha}\right)_0 + (\alpha - \alpha_0)\left(\frac{\partial^2 Q}{\partial \alpha^2}\right)_0 + (\gamma - \gamma_0)\left(\frac{\partial^2 Q}{\partial \gamma \partial \mu}\right)_0$$

$$+ (\mu - \mu_0)\left(\frac{\partial^2 Q}{\partial \alpha \partial \mu}\right)_0 + \text{higher order terms,}$$

where $(\quad)_0$ denotes the value of the function enclosed within the parentheses at the point (α_0, γ_0, μ_0). $\frac{\partial Q}{\partial \gamma}$ and $\frac{\partial Q}{\partial \mu}$ have similar expansions. (Formulas for all of the first and second order partial derivatives of Q are found in an appendix). Setting the left side of each of the expansions to zero (since a point with $\frac{\partial Q}{\partial \alpha} = \frac{\partial Q}{\partial \gamma} = \frac{\partial Q}{\partial \mu} = 0$ is being sought), dropping higher order terms, and rearranging what remain, lead to the system of linear equations

$$(\alpha - \alpha_0)\left(\frac{\partial^2 Q}{\partial \alpha^2}\right)_0 + (\gamma - \gamma_0)\left(\frac{\partial^2 Q}{\partial \alpha \partial \gamma}\right)_0 + (\mu - \mu_0)\left(\frac{\partial^2 Q}{\partial \alpha \partial \mu}\right)_0 = -\left(\frac{\partial Q}{\partial \alpha}\right)_0$$

$$(\alpha - \alpha_0)\left(\frac{\partial^2 Q}{\partial \alpha \partial \gamma}\right)_0 + (\gamma - \gamma_0)\left(\frac{\partial^2 Q}{\partial \gamma^2}\right)_0 + (\mu - \mu_0)\left(\frac{\partial^2 Q}{\partial \gamma \partial \mu}\right)_0 = -\left(\frac{\partial Q}{\partial \gamma}\right)_0$$

$$(\alpha - \alpha_0)\left(\frac{\partial^2 Q}{\partial\alpha\partial\mu} \right)_0 + (\gamma - \gamma_0)\left(\frac{\partial^2 Q}{\partial\gamma\partial\mu} \right)_0 + (\mu - \mu_0)\left(\frac{\partial^2 Q}{\partial\mu^2} \right)_0 = -\left(\frac{\partial Q}{\partial\mu} \right)_0$$

whose solution $(\alpha_1, \gamma_1, \mu_1)$ should be an improved estimate of the point where each of the first order partial derivatives of Q vanishes.

When the above computations are repeated with α_1, γ_1, and μ_1 replacing α_0, γ_0, and μ_0 wherever they occur in the description, a third approximation $(\alpha_2, \gamma_2, \mu_2)$ is obtained. The process can be continued indefinitely. The sequence of approximations $(\alpha_i, \gamma_i, \mu_i)$ for $i=0,1,2,...$, converges to some $(\bar{\alpha}, \bar{\gamma}, \bar{\mu})$ which satisfies the equations

$$\frac{\partial Q}{\partial\alpha} = \frac{\partial Q}{\partial\gamma} = \frac{\partial Q}{\partial\mu} = 0,$$

provided the initial approximation $(\alpha_0, \gamma_0, \mu_0)$ is sufficiently close to $(\bar{\alpha}, \bar{\gamma}, \bar{\mu})$.

In the computer program written for the above computations, the system of linear equations is considered to be in the variables $(\alpha - \alpha_i)$, $(\gamma - \gamma_i)$, and $(\mu - \mu_i)$ (for the ith iteration where $(\ )_i$ replaces $(\ )_0$ for the coefficients of the equations) and the system is solved by Cramer's method. Then α_i, γ_i , and μ_i are added to the solutions to obtain α_{i+1}, γ_{i+1}, and μ_{i+1}. The partial derivatives of Q vanish at extermals other than (local) minima. To insure a correct solution set, the program prints out α_i, γ_i, μ_i, and Q_i after each iteration for the programmer's inspection. If α_{i+1}, γ_{i+1}, μ_{i+1} yield a $Q_{i+1} \leq Q_i$, the program continues to iteration i+2. If $Q_{i+1} > Q_i$, or $Q_{i+1} = Q_i = Q_{i-1}$, then $\alpha = \alpha_i$, $\gamma = \gamma_i$, $\mu = \mu_i$ are taken as the solutions. However, if

$Q_1 > Q_0$, i.e. an increase in Q results on the first refinement of $(\alpha_0, \gamma_0, \mu_0)$, an error message is printed out and computations are halted. In this case new trial values for α_0, γ_0, μ_0 must be entered. Solutions are obtained in about 6 iterations.

After α, γ, and μ have been found, the computer program proceeds to compute and print out the expected cumulative flight hours to accident number n,

$$\varepsilon(t_n) = \sum_{j=0}^{n-1} \frac{1}{\lambda_j}$$

and the accident rate

$$\lambda_n = \alpha + \gamma\mu^n$$

for $n = n_0, 2n_0, 3n_0, \ldots$. ($n_0$ is an input to the program which is chosen to be 10 in the sample case.) Plots of n versus $\varepsilon(t_n)$ and λ_n versus $\varepsilon(t_n)$ should be made and the data points of Table 2 should be superimposed on the plots as a check.

III. APPLICATIONS AND EXTENSIONS OF THE THEORY

The method of analysis that has been presented in this report could be used to evaluate a wide variety of data that relates, or appears to relate, to aircraft accidents. In each case, cumulative flight hours (time) would be used as the independent variable. Various sets of graphs can be compared for indications of trend.

A point of departure for a first analysis of a set of accident data can be the cumulative number of aircraft damaged. The graphs of this preliminary phase for several different models of aircraft may indicate other types of evaluation and comparison that should be conducted. For example, various models of fighter (or bomber or cargo) type aircraft may be compared to see which models have lower accident rates. The conclusions that are drawn from this examination may lead to the scrutinization of still more specific aircraft characteristics, such as single engine versus twin engine, afterburner or no afterburner, and aircraft weight.

The initial sets of data can be broken down into the five aircraft damage classifications as specified in the "Navy Aircraft Accident, Incident, and Ground Accident Reporting Procedure" (OPNAV Inst. 3750.6F of 15 March 1967):

(1) ALFA, alfa damage (destruction or loss)

(2) CHARLIE, substantial damage

(3) DELTA, minor damage

(4) ECHO, limited damage

(5) FOXTROT, no damage.

These cases may point out something about the nature of accidents in which a particular model becomes involved. For instance, it may be indicated that an aircraft is highly prone to accidents resulting in limited damage (ECHO). This result, in turn, may warrant the inspection of the phase of operation in which the accidents happen. Phases of operation could be classified in the following manner:

(1) Static (engines running), incident to flight

(2) Taxiing, incident to flight

(3) Takeoff

(4) Inflight

(5) Landing

(6) Waveoff (go-round)

(7) Nonflight.

For use by the Navy, it may be of particular value to look at the statistics relating to embarked and disembarked aircraft. These statistics could then be broken down into questions about such matters as the type and size of the carrier and the length of time that the carrier has been deployed. An increase in accidents as the time deployed increases may indicate an increase in the operating rate and commitments, over-confidence on the part of pilots and crew, fatigue, and anticipation of the approaching rest and recreation, all of which could be termed unmeasurable factors. Carrier based causes which could be checked might include differences in the methods of carrier operations; carrier operations personnel, such as LSO (landing signal officer), training experience and rating; pitching of the carrier deck; and CVA.

The number of years an aircraft has been in service and the length of time since major overhaul could present some interesting graphs. These could bring into play statistics about material failure, malfunction, quality control, and maintenance procedures. Also, effects of the use of special equipment, machinery, and aircraft support equipment utilized on the aircraft and the ground base may be worth noting.

Other flight related variables that could be considered are the time of day or night of the flight; season of the year; weather conditions including wind, sea state, cloud coverage, ceiling visibility, temperature, and dew point; length of time in flight; and flight altitude and airspeed at the time of the accident.

The pilot being such an important variable in flying may justify comparison of data based on facts like total pilot time, years of military flying, pilot rank, pilot time in the specific aircraft types, pilot time in the last three months, and night pilot time in the last three months.

Another class of accidents that occurs involving aircraft (not incident to flight) is ground accidents with non-aircraft vehicles. Areas for study in this situation might include causes like improper action by the operator, mechanical failure of equipment, improper operator action accompanied by mechanical failure, unforeseen occurrence, and personnel other than the equipment operator.

By plotting several related curves using accumulated aircraft hours as the axis of abscissa will, hopefully, reveal some relationships between people and other aircraft accident factors. These results may then be used in decisions about matters such as aircraft selection, size of aircraft inventory, airframe spares procurement, pilot selection, and carrier operation methods.

IV. ALTERNATE APPROACHED TO THE PROBLEM

The analysis of aircraft accident information is of great and immediate importance to the U. S. Navy, as well as the U. S. Air Force. Studies are constantly being conducted to find ways to evaluate the available data in order to come to conclusions and decisions concerning non-combat aircraft accidents.

Some of the studies and evaluations conducted are discussed here to show other approaches that have been used in the analysis of aircraft accident data.

The use of plots of cumulative accidents as they occurred in time in aircraft accident analysis is well known. If the curve for the given values closely approximates a straight line, the accident rate is constant (see Figure 3). When the accident rate is a constant, i.e., the hazard function given by:

$$h(t) = \frac{f(t)}{1 - F(t)}$$

is constant, say $h(t) = \lambda$, then

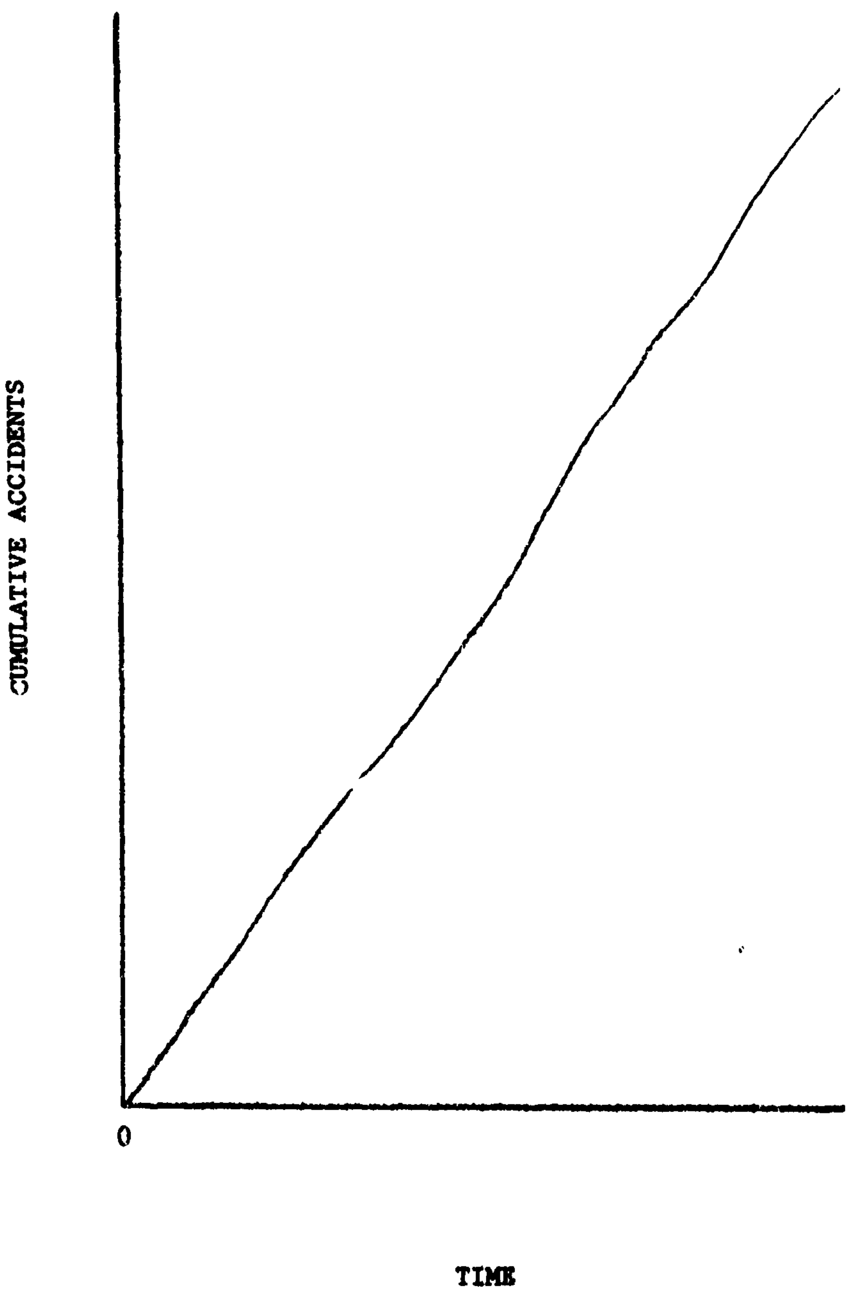
$$\begin{aligned} f(t) &= h(t)\,[1 - F(t)] \\ &= h(t)\exp[\,-\textstyle\int_0^t h(t)\,dt\,]\,. \end{aligned}$$

Denote the slope of the graph by λ, so the density function becomes

$$\begin{aligned} f(t) &= \lambda\exp[-\textstyle\int_0^t \lambda dt] \\ &= \lambda\exp\{-\,[\lambda t]_0^t\,\} \\ &= \lambda\exp(-\lambda t). \end{aligned}$$

FIGURE 3

CUMULATIVE ACCIDENTS AS THEY OCCURRED IN TIME



f(t) represents the exponential probability function where λ is the rate of accident occurrence and $1/\lambda$ is the mean time to occurrence.

The maximum likelihood estimator for the sample occurrence rate is normally distributed about the true rate, since the distribution of the sample mean is asymptotically normal as the sample size approaches infinity. A confidence interval about the true mean is needed since the true variance is not known. Using

$$t = \frac{\sqrt{n-1}\,(\hat{\lambda} - \lambda)}{S}$$

where t is distributed with (n-1) degrees of freedom, $\hat{\lambda}$ is the sample mean, S is the sample standard deviation, λ is the true accident rate, and n is the number of accidents, then

$$\hat{\lambda} \pm \frac{t_{\alpha/2} \cdot S}{\sqrt{n-1}}$$

gives a $(1-\alpha)$ confidence interval for the true aircraft accident rate λ.

Another method of attacking the accident prediction problem involves the systematic investigation and evaluation of statistical information in order to derive a single equation (or perhaps a set of equations) which can be used to estimate aircraft attrition.

This is the process used by T.E. Anger in The Estimation of Peacetime Aircraft Attrition, a Center for Naval Analyses research contribution. The variables Mr. Anger felt affected total attrition of forces of aircraft included:

(1) Total flying hours of aircraft force

(2) Proportion of total flying done from carriers

(3) Empty weight

(4) Maximum speed

(5) Number of engines.

From the results of statistical measures, the variables listed above are incorporated into an attrition-estimating equation (the actual equation is classified).

W.E. Mooz assumes that the cumulative number of aircraft destroyed is a function of the cumulative number of flying hours (dependent variable) in his Rand Corporation Memorandum <u>Relationships for Estimating Peacetime Aircraft Attrition</u>. This function appears as a reasonably straight line on log-log paper, and therefore demonstrates that the cumulative number of aircraft destroyed is a decreasing exponential function of the flying activity of the aircraft. Mooz states this is "evidence that the attrition pattern of a given type and model of aircraft was subject to an orderly learning process which continues throughout its flying life". The exponential function $y=ax^b$ is used to approximate the accident curve.

According to this study conducted by Rand, the quarterly attrition rate is not as useful a variable as many people think. Quarterly attrition rates do not involve equal flying times, and the number of hours flown varies for the different models. Since flying programs differ, there can be no true comparison between quarterly attrition rates, and also attrition does vary over the flying life of the aircraft. To summarize, Mooz says that "attrition rate is awkward from a statistical standpoint".

A general conclusion is made by A. J. Gross and Milton Kamins in Reliability Assessment in the Presence of Reliability Growth. They show that there is no evidence that would lead to the generalization that the age of any fighter should result in any increase in accident rates, provided normal preventive maint‘ ‿nce and product improvement are continued.

Three statistical examinations of accident and attrition data for jet fighters is presented and compared by Milton Kamins in Jet Fighter Accident/Attrition Rates in Peacetime: An Application of Reliability Growth Modelling. In the first model, the reliability, R_k, at any stage k of the process is given by

$$R_k = R_\infty - \frac{c}{k}$$

where R_∞ is the ultimate reliability and c is the total amount of reliability growth that can be achieved from stage 1 to stage infinity (i.e., $R_\infty - R_1$). Accident rate (unreliability) can be treated as the complement of accident reliability which is very close to 1.0. Accident rate, F, is expressed as events per hundred thousand flying hours or hundred thousands landings.

$$1 - F_k = 1 - F_\infty - \frac{c}{k}$$

where

$$c = F_1 - F_\infty$$

so

$$F_k = F_\infty + \frac{c}{k} .$$

Let $F_\infty \to 0$

$$\log F_k = \log c - \log k$$

and

$$c = F_1 .$$

Therefore

$$\log F_k = \log F_1 - \log k.$$

Maximum likelihood estimates are then developed for the parameters F_∞ and c in this hyperbolic model of reliability growth. For cases evaluated by this model, landings rather than flying hours are used as a measure of exposure, since they appear to be a better indicator of risk. Where material failure is a contributing cause to accidents, the accident rate seems to be a function of the number of years in service. In order to cause the model to appear as a straight line, the independent variable is the reciprocal of the number of years in service (see Figures 4A and 4B). With this type of graph, comparisons can easily be made between aircraft which involve carry-over technology and experience. Comparison of these figures A and B show that even though the ultimate accident rate, F_∞ , is near zero for both aircraft, it is sufficiently higher in B than in A to cause the curves to cross at about the 8-year point. Thus it is shown that even though there is improvement with calendar

FIGURE 4A

AIRCRAFT ACCIDENTS AS A FUNCTION OF THE NUMBER OF YEARS IN SERVICE

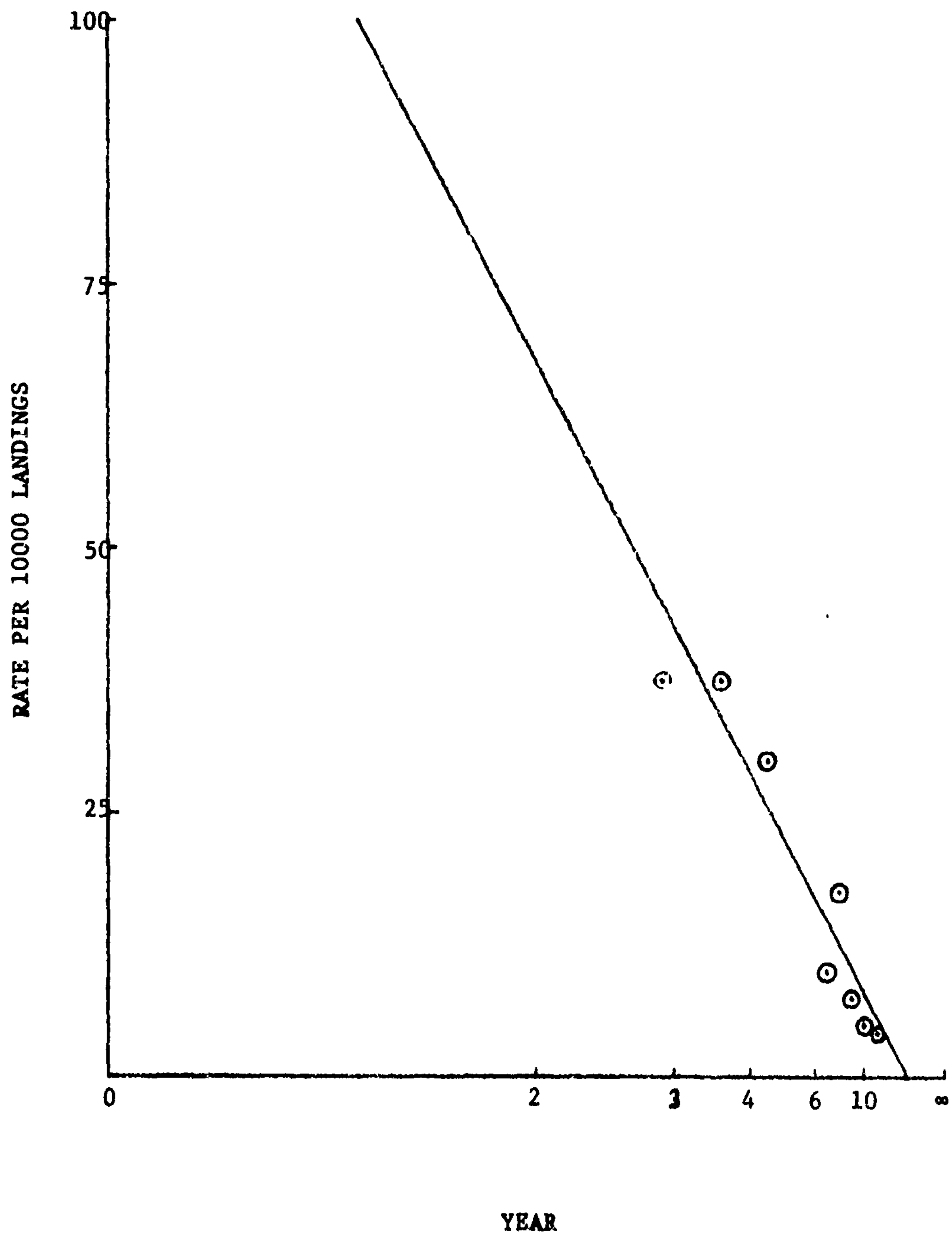
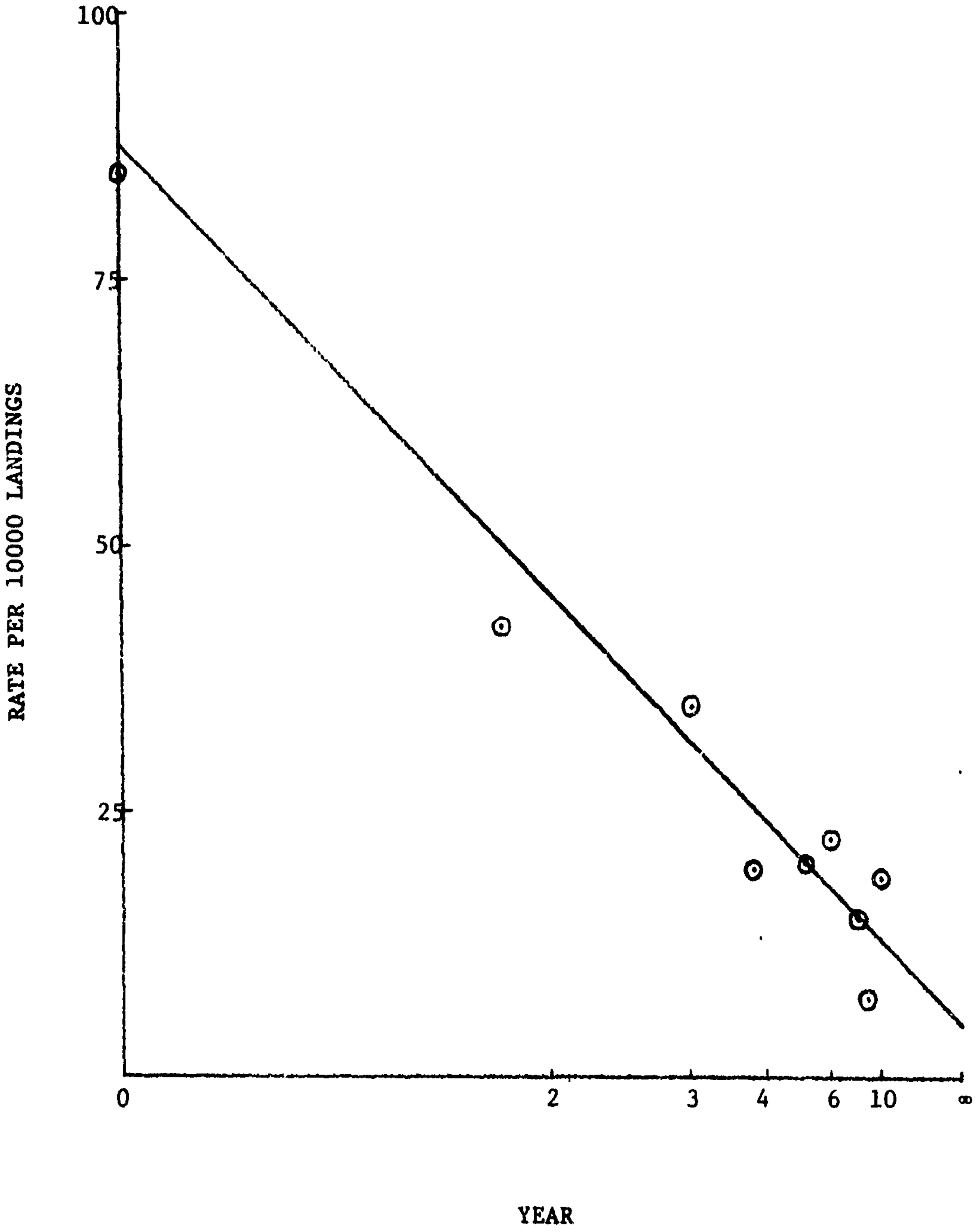


FIGURE 4B

AIRCRAFT ACCIDENTS AS A FUNCTION OF THE NUMBER OF YEARS IN SERVICE



time, the second aircraft becomes more prone to material failure accidents. Other comparisons can be made to indicate material reliability, such as the fraction of all accidents in which material failure is a factor by years. The same graphs and evaluations can also be made for all causes of accidents.

The second model of reliability growth examined is the (negative) exponential model in which the reliability R_k at stage k of a development process can be expressed as:

$$R_k = 1 - \alpha e^{-\beta k}$$

α represents the total amount of growth that can be achieved from stage zero to stage infinity, and β is a measure of the rate of growth.

A third model, called a learning curve, states that the total number of accidents (or losses) L_x depends on the total usage x (flying hours or landings).

$$L_x = AxB$$

where A is the risk for the first flying hour or landing, and B is the complement of the rate of learning. The parameters are estimated by a weighted least-squares technique.

Kamins' three models are compared on the basis of a chi-square goodness of fit test of the observed data by year against the predictions of each model with parameters estimated from the data. Results of the evaluations and comparisons of the three models show that the hyperbolic model of reliability growth is preferable for three reasons. It seems to represent trends more accurately, is easier to use, and useful confidence limits for past experience or future projections of accident or attrition rates can be calculated.

John M. Cozzolino deals with the infant mortality effect of statistical reliability theory in a paper titled Probabilistic Models of Decreasing Failure Rate Processes. Infant mortality implies a decrease of the conditional probability of failure of a device with increases in age. Cozzolino compares his four models of decreasing failure rate processes based upon the population heterogeneity hypothesis and incorporating explicit repair assumptions (see Table 3).

The various models presented could be used in predicting different aspects of aircraft accidents. For instance, the component variability model could be used in predicting the aircraft accidents which will result in total destruction (non-repairable).

All the methods and models that have been described have some theoretical application to the case of aircraft accident prediction. Each has its advantages and disadvantages, so it is necessary to choose a method (or methods) that most accurately fits the situation being evaluated.

TABLE 3

COMPARISONS OF FOUR MODELS OF DECREASING FAILURE RATE PROCESSES

MODEL	CAUSE of D.F.R.	NATURE OF REPAIR: FRACTION REPLACED	NATURE OF REPAIR: QUALITY OF REPLACEMENT	FUNCTIONAL FORM OF FAILURE RATE	FAILURE RATE JUMP WITH FAILURE
Component Variability	Variable Quality of Component Parts	100%	Imperfect (new)	Hyperbolic	Up to Initial Value
Initial Defects	Variable Quality of Production Process	0%	Perfect	Exponential	Unaffected
N-Component Variable	Variable Quality of Component Parts	100% N	Imperfect (new)	Hyperbolic	Up an Intermediate Amount
Time Accumulation	Variable Quality of Component Parts	100% N	Imperfact (new)	Hyperbolic	Up an Intermediate Amount

V. COMPUTER PROGRAMS

The following are listings of the two programs used in this analysis.

The first program finds a linear solution to a set of equations by Cramer's Rule. The second program modifies the input data, minimizes Q, and determines the parameters α, γ, and μ.

LINEAR PROGRAM LISTING

```
      SUBROUTINE LINEAR
      COMMON A(3,4),S(3)
      DIMENSION B(3,4),DET(4)
      DO 1Ø I=1,3
      DO 1Ø J=1,4
   1Ø B(I,J) = A(I,J)
      DO 6Ø N=1,4
      DO 2Ø I=1,3
   2Ø B(I,N) = A(I,4)
      I = 1
      J = 2
      K = 3
      R = 1.
      DET(N) = Ø.
      DO 5Ø L=1,6
      DET(N) = DET(N)+R*B(1,I)*B(2,J)*B(3,K)
      JJ = J
      IF (R) 3Ø,3Ø,4Ø
   3Ø J = K
      K = JJ
      GO TO 5Ø
   4Ø J = I
      I = JJ
   5Ø R = -R
      DO 6Ø I=1,3
   6Ø B(I,N) = A(I,N)
      DO 7Ø I=1,3
   7Ø S(I) = DET(I)/DET(4)
      RETURN
      END
```

MAIN PROGRAM LISTING

```
      COMMON QAA,DQAG,DQAM,QAG,QGG,DQGM,QAM,QGM,QMM,SA,SG,SM,ALA,GAM,MU
      DIMENSION ACT(100),CN(100),CT(100),E(100),EA(100),EAA(100),
     1EAG(100),EAM(100),EG(100),EGG(100),EGM(100),EGM1(100),EGM2(100),
     2EM(100),EMM(100),EMM1(100),EMM2(100),RATE(100),T(100),TI(100)
      REAL IACT,ILAM,JMJ,JMJ2,J1JMJ,J2MJ2,LAC,LAJ,LAJ2,LAJ3,LAM(100),
     1LAO,LMC,MC,MJ,MJ2,MO,MU,M1,N(100),NI(100)
      DO 460 NCASE=1,4
      WRITE (6,10) NCASE
   10 FORMAT (////,32X,5HCASE ,I1)
      TEN4 = 10.**(-4)
      READ (5,20) IM
   20 FORMAT (I6)
      I = 1
      TT = 0.
      DO 90 J=1,IM
      READ (5,30) T(J),N(J)
   30 FORMAT (2F10.0)
      TI(J) = T(J)
      NI(J) = N(J)
      IF (N(J)) 40,70,80
   40 WRITE (6,50)
   50 FORMAT (/,28H N(K) SHOULD NOT BE NEGATIVE)
      WRITE (6,60) K,N(K)
   60 FORMAT (10X,4HK = ,I6,4X,7HN(K) = ,F6.0)
      GO TO 460
   70 TT = TT+T(J)
      GO TO 90
   80 T(I) = (TT+T(J))*TEN4
      N(I) = N(J)
      RATE(I) = N(I)/T(I)
      I = I+1
      TT = 0.
   90 CONTINUE
      M = I-1
      CT(1) = T(1)
      CN(1) = N(1)
      DO 100 I=2,M
      CT(I) = CT(I-1)+T(I)
  100 CN(I) = CN(I-1)+N(I)
      WRITE (6,105)
  105 FORMAT (//,10X,10HINPUT DATA,17X,25HMODIFIED CUMULATIVE INPUT,
     15H DATA)
      WRITE (6,110)
  110 FORMAT (/,4X,6HNUMBER,8X,8HAIRCRAFT,8X,6HNUMBER,5X,9HAIRCRAFT ,
     15HHOURS,4X,8HACCIDENT)
      WRITE (6,115)
  115 FORMAT (13H OF ACCIDENTS,7X,5HHOURS,6X,12HOF ACCIDENTS,4X,
     110HX 10**(-4),8X,4HRATE)
      DO 120 I=1,M
  120 WRITE (6,130) NI(I),TI(I),CN(I),CT(I),RATE(I)
```

```
130 FORMAT (3X,F6.0,7X,F9.0,8X,F6.0,8X,F9.4,5X,F9.4)
    IF (MM-M) 137,137,133
133 MM1 = M+1
    DO 135 I=MM1,MM
135 WRITE (6,130) MI(I),TI(I)
137 IF (M-3) 140,160,160
140 WRITE (6,150) M
150 FORMAT (/,35H MODIFIED INPUT DATA HAS ONLY ,I1,
   129H PERIODS WHICH IS LESS THAN 3)
    GO TO 460
160 WRITE (6,170)
170 FORMAT (/,52H INPUT INITIAL AND FINAL RATES AND INTERMEDIATE RAT
   115H AND CUMULATIVE)
    WRITE (6,175)
175 FORMAT (29H ACCIDENTS USING FORMAT 4F5.1)
    READ (9,180) LA0,AL0,LAC,CNA
180 FORMAT (4F5.1)
    GA0 = LAC-AL0
    MC = (LAC-AL0)/GA0
    LMC = ALOG(MC)/CNA
    M0 = EXP(LMC)
    WRITE (6,200)
200 FORMAT (//,20X,25HSUCCESSIVE APPROXIMATIONS)
    WRITE (6,210)
210 FORMAT (/,8X,5HALPHA,12X,5HGAMMA,11X,2HMU,12X,1HM)
220 IEND = 0
    JPASS = 0
    AL1 = 0.
    GA1 = 0.
    M1 = 0.
    Q = 999999999.
230 Q1 = Q
    DO 260 K=1,M
    J = -1
    E(K) = 0.
    EA(K) = 0.
    EG(K) = 0.
    EM(K) = 0.
    EAA(K) = 0.
    EGG(K) = 0.
    EMM1(K) = 0.
    EMM2(K) = 0.
    EAG(K) = 0.
    EAM(K) = 0.
    EGM1(K) = 0.
    EGM2(K) = 0.
240 J = J+1
    MJ = M0**J
    MJ2 = MJ*MJ
    JMJ = J*MJ
```

```
      JMJ2 = J*MJ2
      J2MJ2 = J*JMJ2
      J1JMJ = (J-1)*JMJ
      LAJ = ALO+GAO*MJ
      LAJ2 = LAJ*LAJ
      LAJ3 = LAJ*LAJ2
      E(K) = E(K)+1./LAJ
      EA(K) = EA(K)+1./LAJ2
      EG(K) = EG(K)+MJ/LAJ2
      EM(K) = EM(K)+JMJ/LAJ2
      EAA(K) = EAA(K)+1./LAJ3
      EGG(K) = EGG(K)+MJ2/LAJ3
      EMM1(K) = EMM1(K)+J2MJ2/LAJ3
      EMM2(K) = EMM2(K)+J1JMJ/LAJ2
      EAG(K) = EAG(K)+MJ/LAJ3
      EAM(K) = EAM(K)+JMJ/LAJ3
      EGM1(K) = EGM1(K)+JMJ2/LAJ3
      EGM2(K) = EGM2(K)+JMJ/LAJ2
      IF (J-CN(K)+1) 24Ø,25Ø,24Ø
  25Ø E(K) = E(K)-CT(K)
      EA(K) = -EA(K)
      EG(K) = -EG(K)
      EM(K) = (-GAO/MO)*EM(K)
      EAA(K) = 2.*EAA(K)
      EGG(K) = 2.*EGG(K)
      EMM(K) = ((2.*GAO*GAO)/(MO*MO))*EMM1(K)-(GAO/(MO*MO))*EMM2(K)
      EAG(K) = 2.*EAG(K)
      EAM(K) = ((2.*GAO)/MO)*EAM(K)
  26Ø EGM(K) = ((2.*GAO)/MO)*EGM1(K)-(1./MO)*EGM2(
      Q = Ø.
      QA = Ø.
      QG = Ø.
      QM = Ø.
      QAA = Ø.
      QGG = Ø.
      QMM = Ø.
      QAG = Ø.
      QAM = Ø.
      QGM = Ø.
      DO 27Ø K=1,N
      Q = Q+E(K)*E(K)
      QA = QA+E(K)*EA(K)
      QG = QG+E(K)*EG(K)
      QM = QM+E(K)*EM(K)
      QAA = QAA+EA(K)*EA(K)+E(K)*EAA(K)
      QGG = QGG+EG(K)*EG(K)+E(K)*EGG(K)
      QMM = QMM+EM(K)*EM(K)+E(K)*EMM(K)
      QAG = QAG+EA(K)*EG(K)+E(K)*EAG(K)
      QAM = QAM+EA(K)*EM(K)+E(K)*EAM(K)
  27Ø QGM = QGM+EG(K)*EM(K)+E(K)*EGM(K)
```

```
      C = 0.5*C
      IEND = IEND+1
      WRITE (6,280) ALC,GAC,MC,C
  280 FORMAT (4F15.8)
      IF (C-C1) 330,290,350
  290 JPASS = JPASS+1
      IF (ABS(AL1-ALC)+ABS(GA1-GAC)+ABS(M1-MC)) 300,310,300
  300 IF (JPASS-3) 340,310,340
  310 IF (IEND-3) 320,320,350
  320 ALC = ALC+TEMP
      GAC = GAC+TEMP
      MC = MC-TEMP
      GO TO 220
  330 JPASS = 0
  340 SA = -CA
      SG = -CG
      SM = -CM
      DCAG = CAG
      DCAM = CAM
      DCGM = CGM
      CALL LINEAR
      AL1 = ALC
      GA1 = GAC
      M1 = MC
      ALC = ALC+ALA
      GAC = GAC+GAM
      MC = MC+MU
      GO TO 230
  350 IF (IEND-2) 360,440,360
  360 ALA = AL1
      GAM = GA1
      MU = M1
      ILAM = ALA+GAM
      IACT = 1./ILAM
      NN = CM(M)+10.
      K = 1
      L = 10
      DO 380 J=2,NN
      ILAM = ALA+GAM*MU**(J-1)
      IACT = IACT+1./ILAM
      IF (J-L) 380,370,380
  370 LAM(K) = ILAM
      ACT(K) = IACT
      K = K+1
      L = L+10
  380 CONTINUE
      WRITE (6,390)
  390 FORMAT (//,19X,11HOUTPUT DATA,/)
      WRITE (6,400) ALA,GAM,MU
  400 FORMAT (9H ALPHA = ,F12.8,2X,8HGAMMA = ,F12.8,2X,5HMU = ,F12.8)
```

```
      WRITE (6,41Ø)
  41Ø FORMAT (/,11X,1ØHCUMULATIVE)
      WRITE (6,412)
  412 FORMAT (4X,6HNUMBER,8X,14HAIRCRAFT HOURS,4X,8HACCIDENT)
      WRITE (6,415)
  415 FORMAT (13H OF ACCIDENTS,7X,1ØHX 1Ø**(-4),8X,4HRATE)
      K = K-1
      L = 1Ø
      DO 43Ø J=1,K
      WRITE (6,42Ø) L,ACT(J),LAM(J)
  42Ø FORMAT (2X,I6,12X,F9.4,5X,F9.4)
  43Ø L = L+1Ø
      GO TO 46Ø
  44Ø WRITE (6,45Ø)
  45Ø FORMAT (/,48H TRY ANOTHER METHOD FOR OBTAINING INITIAL VALUES)
  46Ø CONTINUE
  47Ø STOP
      END
/DATA
```

VI. SAMPLE TEST CASES

Since Navy accident data is classified, it has been necessary to set-up sample statistical information that will serve as a set of representative cases.

Included in this section are a listing of the accident data used, the computer print out for the four sample cases, and the graphs that relate to each case.

TEST CASE DATA

32

0.	0.
1693.	4.
2545.	5.
3510.	3.
4361.	3.
5205.	4.
6490.	5.
8059.	6.
10306.	5.
7380.	4.
14255.	5.
14063.	7.
14890.	7.
16256.	5.
16400.	5.
17174.	6.
19668.	5.
20679.	6.
18686.	6.
17316.	8.
19910.	6.
20983.	10.
39898.	12.
42016.	12.
43122.	9.
49364.	15.
42673.	11.
43177.	11.
36686.	6.
42611.	10.
39322.	8.
35735.	14.

32

0.	0.
1693.	2.
2545.	3.
3510.	1.
4361.	1.
5205.	2.
6490.	3.
8059.	3.
10306.	2.
7380.	3.
14255.	2.
14063.	3.
14890.	4.
16256.	3.

164ØØ.	2.
17174.	4.
19668.	4.
2Ø679.	3.
18686.	4.
17316.	6.
1991Ø.	3.
2Ø983.	5.
39898.	8.
42Ø16.	7.
43122.	8.
49364.	8.
42673.	1Ø.
43177.	7.
36686.	5.
42611.	7.
39322.	6.
35735.	9.
32	
64.	1.
3.	Ø.
13Ø.	2.
334.	7.
681.	4.
1827.	4.
973.	1.
719.	1.
143Ø.	3.
1512.	2.
1726.	3.
251Ø.	4.
329Ø.	4.
3221.	4.
2247.	4.
2768.	4.
3385.	5.
4197.	5.
399Ø.	5.
45Ø9.	7.
5512.	5.
9168.	13.
8676.	11.
1Ø285.	12.
12499.	6.
11315.	15.
12271.	12.
15Ø88.	13.
6869.	3.
37Ø2.	7.
7Ø83.	11.

5482.	Ø.
32	
64.	Ø.
3.	Ø.
13Ø.	Ø.
334.	2.
681.	1.
1827.	1.
973.	Ø.
719.	Ø.
143Ø.	1.
1512.	1.
1726.	1.
251Ø.	2.
329Ø.	1.
3221.	1.
2247.	2.
2768.	1.
3385.	2.
4197.	1.
399Ø.	2.
45Ø9.	3.
5512.	2.
9168.	5.
8676.	5.
1Ø285.	3.
12499.	1.
11315.	4.
12271.	3.
15Ø88.	6.
6869.	2.
3792.	2.
7Ø83.	5.
5482.	Ø.

CASE 1

INPUT DATA		MODIFIED CUMULATIVE INPUT DATA		
NUMBER OF ACCIDENTS	AIRCRAFT HOURS	NUMBER OF ACCIDENTS	AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
0.	0.	4.	0.1693	23.6267
4.	1693.	9.	0.4238	19.6464
5.	2545.	12.	0.7748	8.5470
3.	3510.	15.	1.2109	6.8792
3.	4361.	19.	1.7314	7.6849
4.	5205.	24.	2.3804	7.7042
5.	6490.	30.	3.1863	7.4451
6.	8059.	35.	4.2169	4.8515
5.	10300.	39.	4.9549	5.4201
4.	7380.	44.	6.3804	3.5075
5.	14255.	51.	7.7867	4.9770
7.	14963.	58.	9.2757	4.7011
7.	14890.	63.	10.9013	3.0758
5.	16256.	68.	12.5413	3.0488
5.	16400.	74.	14.2587	3.4937
6.	17176.	79.	16.2255	2.5422
5.	19008.	85.	18.2934	2.9015
6.	20079.	91.	20.1620	3.2110
6.	18086.	99.	21.8930	4.6200
8.	17316.	105.	23.8840	3.0130
6.	19910.	115.	25.9828	4.7658
10.	20983.	127.	29.9720	3.0077
12.	39898.	139.	34.1742	2.8501
12.	42016.	148.	38.4864	2.0871
9.	43122.	163.	43.4228	3.0387
15.	49364.	174.	47.6901	2.5777
11.	42073.	185.	52.0078	2.5477
11.	43177.	191.	55.0764	1.6355
6.	36086.	201.	59.9375	2.3468
10.	42611.	209.	63.8697	2.0345
8.	39322.	223.	67.4432	3.9177
14.	35735.			

INPUT INITIAL AND FINAL RATES AND INTERMEDIATE RATE AND CUMULATIVE
ACCIDENTS USING FORMAT 4F5.1
H.0077 ENTER DATA.
11.5 2.0 3.0 100.0

SUCCESSIVE APPROXIMATIONS

ALPHA	GAMMA	MU	C
2.00000000	9.50000000	0.97773802	50.57025146
2.13074589	7.03795528	0.98045224	18.82631220
2.17228889	6.26936626	0.98208463	10.80158752
1.85422230	4.97937107	0.98883957	27.12139893

OUTPUT DATA

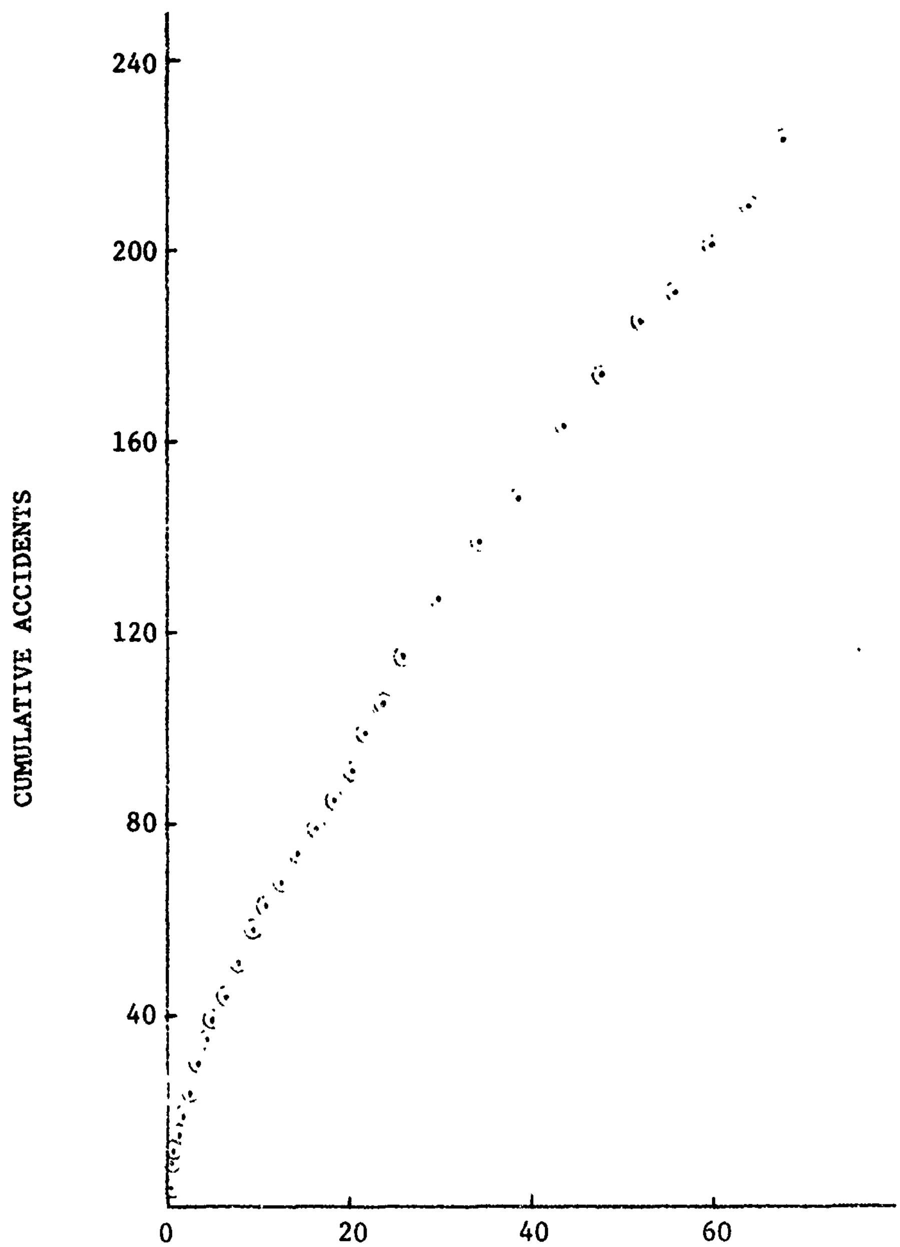
ALPHA = 2.17228889 GAMMA = 6.26936626 MU = 0.98208463

NUMBER OF ACCIDENTS	CUMULATIVE AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
10	1.2581	7.5003
20	2.6880	6.6191
30	4.3018	5.8837
40	6.1096	5.2699
50	8.1191	4.7576
60	10.3350	4.3301
70	12.7587	3.9732
80	15.3881	3.6754
90	18.2181	3.4268
100	21.2405	3.2193
110	24.4448	3.0402
120	27.8186	2.9016
130	31.3482	2.7810
140	35.0194	2.6804
150	38.8177	2.5963
160	42.7291	2.5262
170	46.7402	2.4677
180	50.8385	2.4188
190	55.0125	2.3781
200	59.2519	2.3440
210	63.5475	2.3156
220	67.8911	2.2919
230	72.2755	2.2721

FIGURE 5

CASE 1

ACTUAL AIRCRAFT ACCIDENT DATA



CUMULATIVE AIRCRAFT HOURS X 10^{-4}

FIGURE 6

CASE 1

ACTUAL CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE

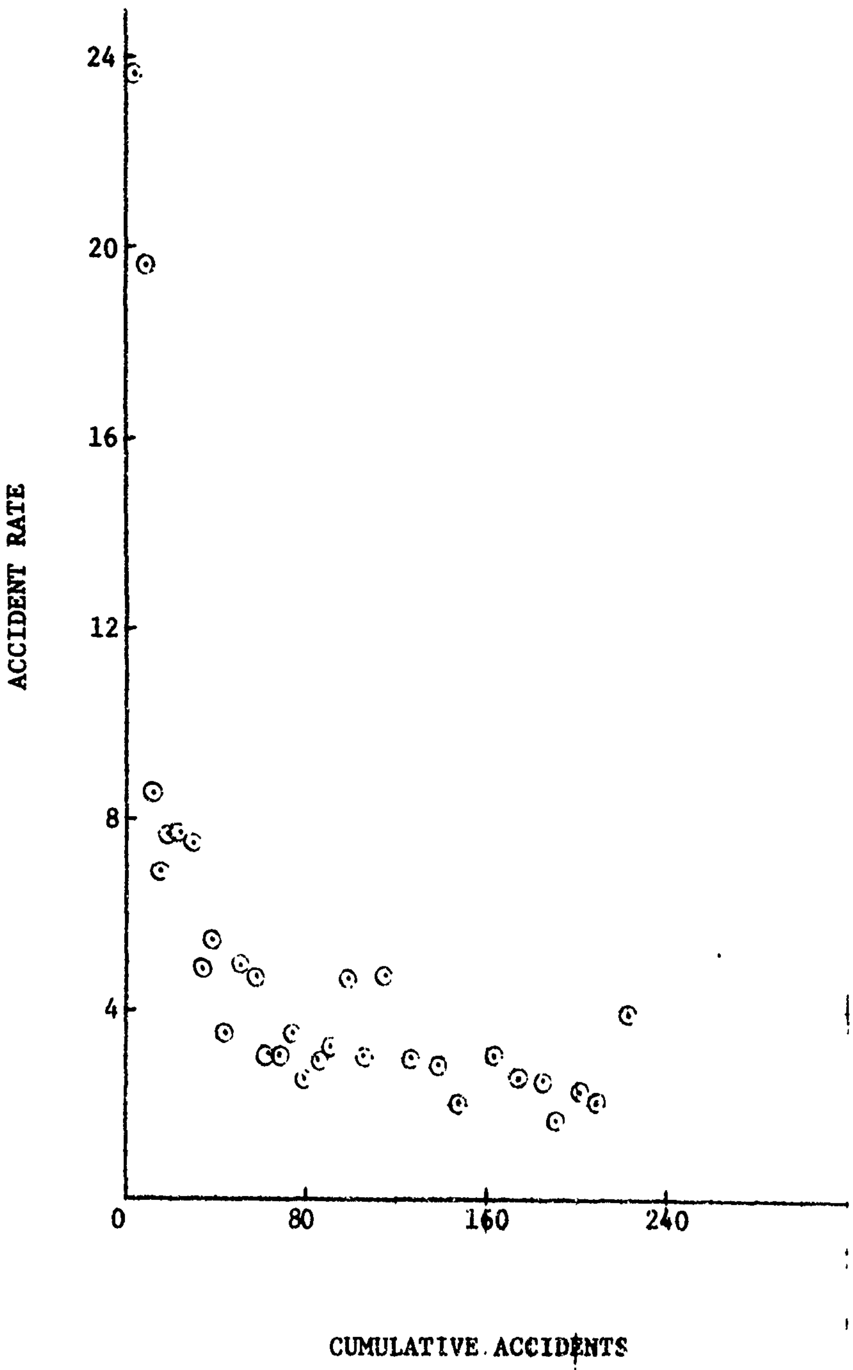


FIGURE 7

CASE 1

APPROXIMATE AIRCRAFT ACCIDENT CURVE

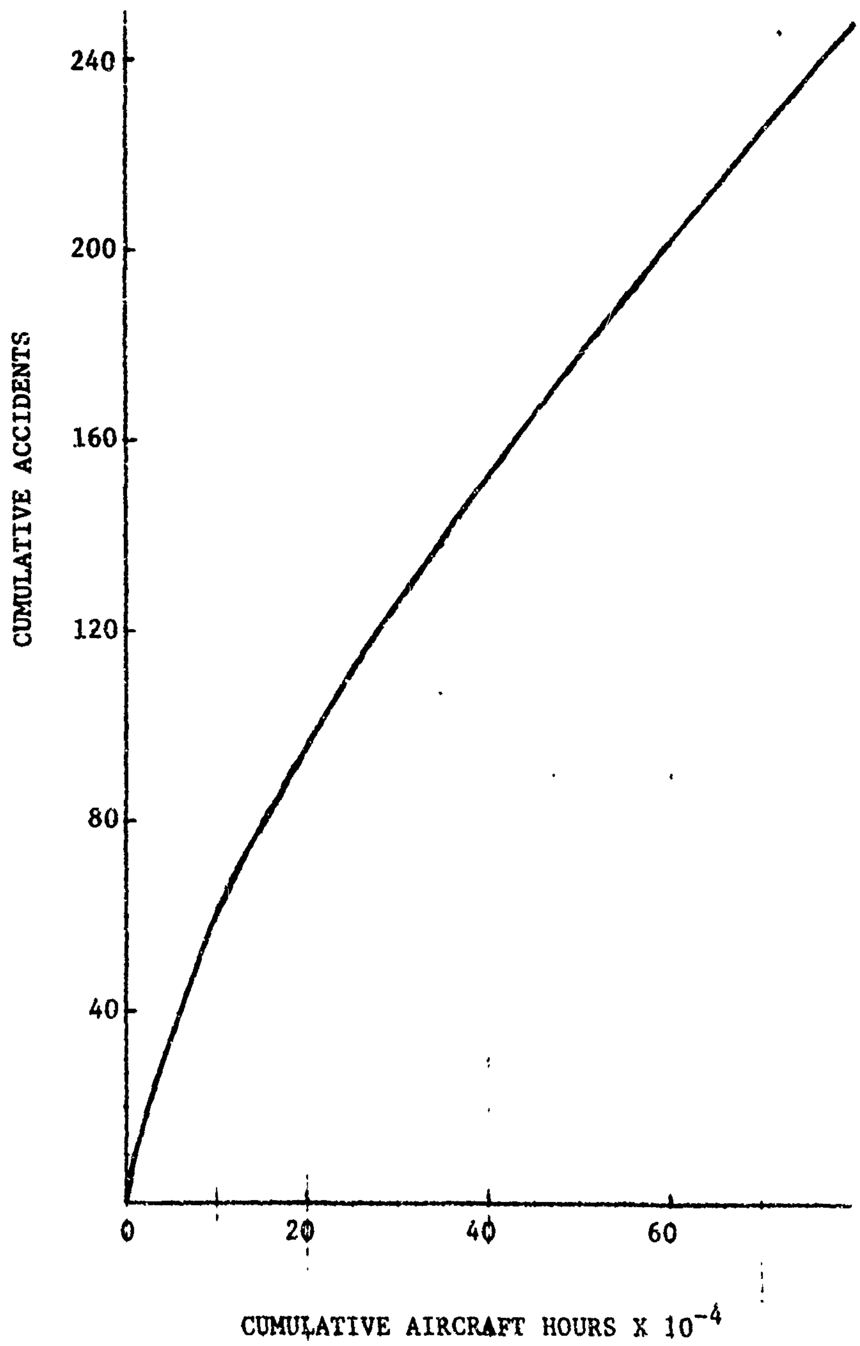
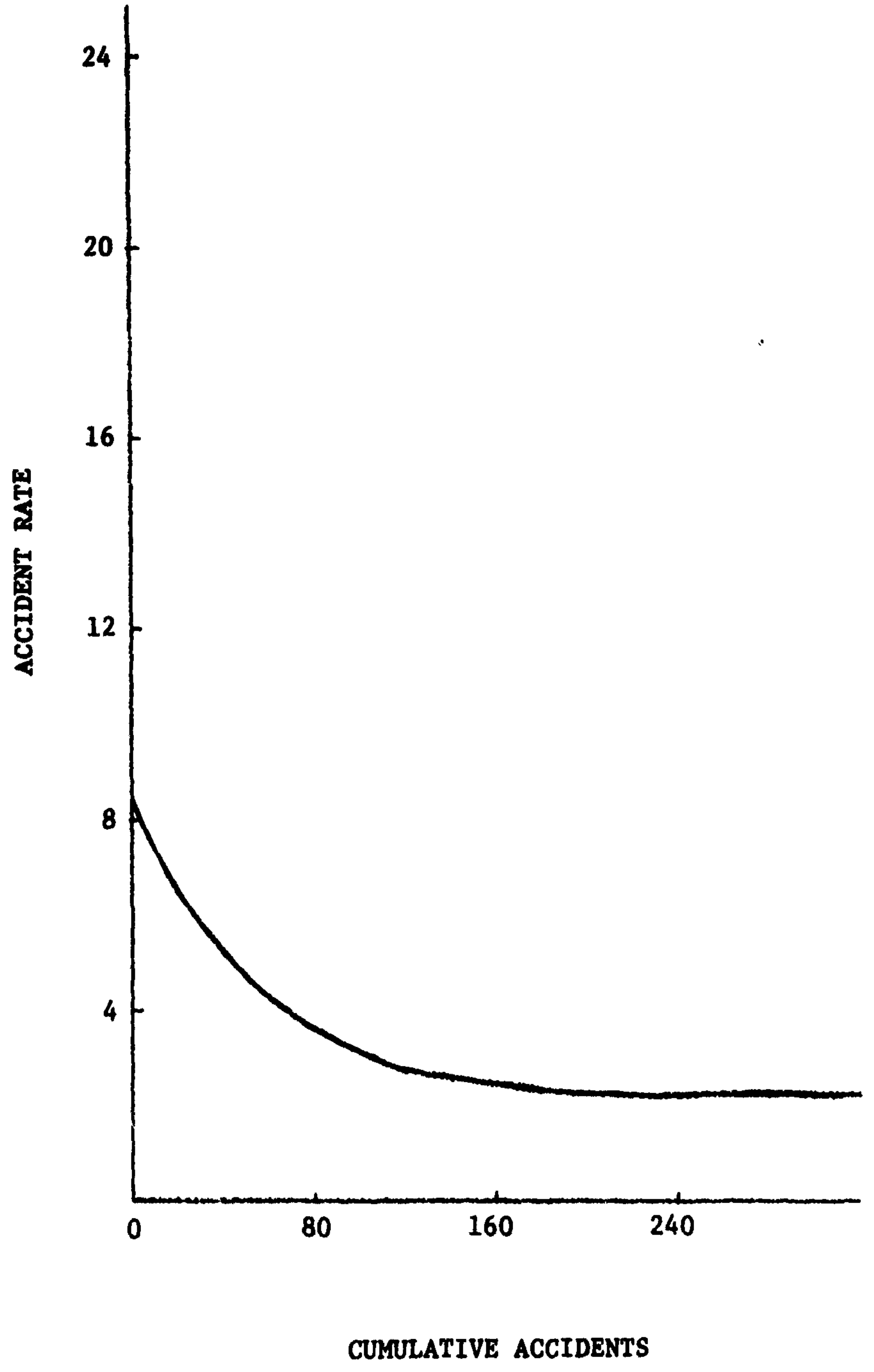


FIGURE 8

CASE 1

APPROXIMATE CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE



CASE 2

INPUT DATA		MODIFIED CUMULATIVE INPUT DATA		
NUMBER OF ACCIDENTS	AIRCRAFT HOURS	NUMBER OF ACCIDENTS	AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
0.	0.	2.	0.1693	11.8134
2.	1693.	5.	0.4238	11.7878
3.	2545.	6.	0.7748	2.8490
1.	3510.	7.	1.2109	2.2931
1.	4361.	9.	1.7314	3.8425
2.	5205.	12.	2.3804	4.6225
3.	6490.	15.	3.1863	3.7225
3.	8059.	17.	4.2169	1.9406
2.	10306.	20.	4.9549	4.0650
3.	7380.	22.	6.3804	1.4030
2.	14255.	25.	7.7807	2.1333
3.	14063.	29.	9.2757	2.6864
4.	14890.	32.	10.9013	1.8455
3.	16250.	34.	12.5413	1.2195
2.	16400.	36.	14.2587	2.3291
4.	17174.	42.	16.2255	2.0338
4.	19668.	45.	18.2934	1.4507
3.	20679.	49.	20.1620	2.1406
4.	18686.	55.	21.8936	3.4650
6.	17316.	58.	23.8846	1.5068
3.	19910.	63.	25.9828	2.3829
5.	20983.	71.	29.9726	2.0051
8.	39898.	78.	34.1742	1.6660
7.	42016.	86.	38.4804	1.8552
8.	43122.	94.	43.4228	1.6206
8.	49364.	104.	47.6901	2.3434
10.	42673.	111.	52.0078	1.6212
7.	43177.	116.	55.6764	1.3629
5.	36686.	123.	59.9375	1.6428
7.	42611.	129.	63.8697	1.5259
6.	39322.	138.	67.4432	2.5185
9.	35735.			

INPUT INITIAL AND FINAL RATES AND INTERMEDIATE RATE AND CUMULATIVE
ACCIDENTS USING FORMAT 4F5.1
M.0077 ENTER DATA.
4.5 1.5 2.0 50.0

SUCCESSIVE APPROXIMATIONS

ALPHA	GAMMA	MU	C
1.50000000	3.00000000	0.96479928	34.07690533
1.58477497	2.00770378	0.97043335	15.92052460
1.59876156	1.94672012	0.97252291	11.42836761
3.48464584	4.59255791	0.83409184	2498.21679087

OUTPUT DATA

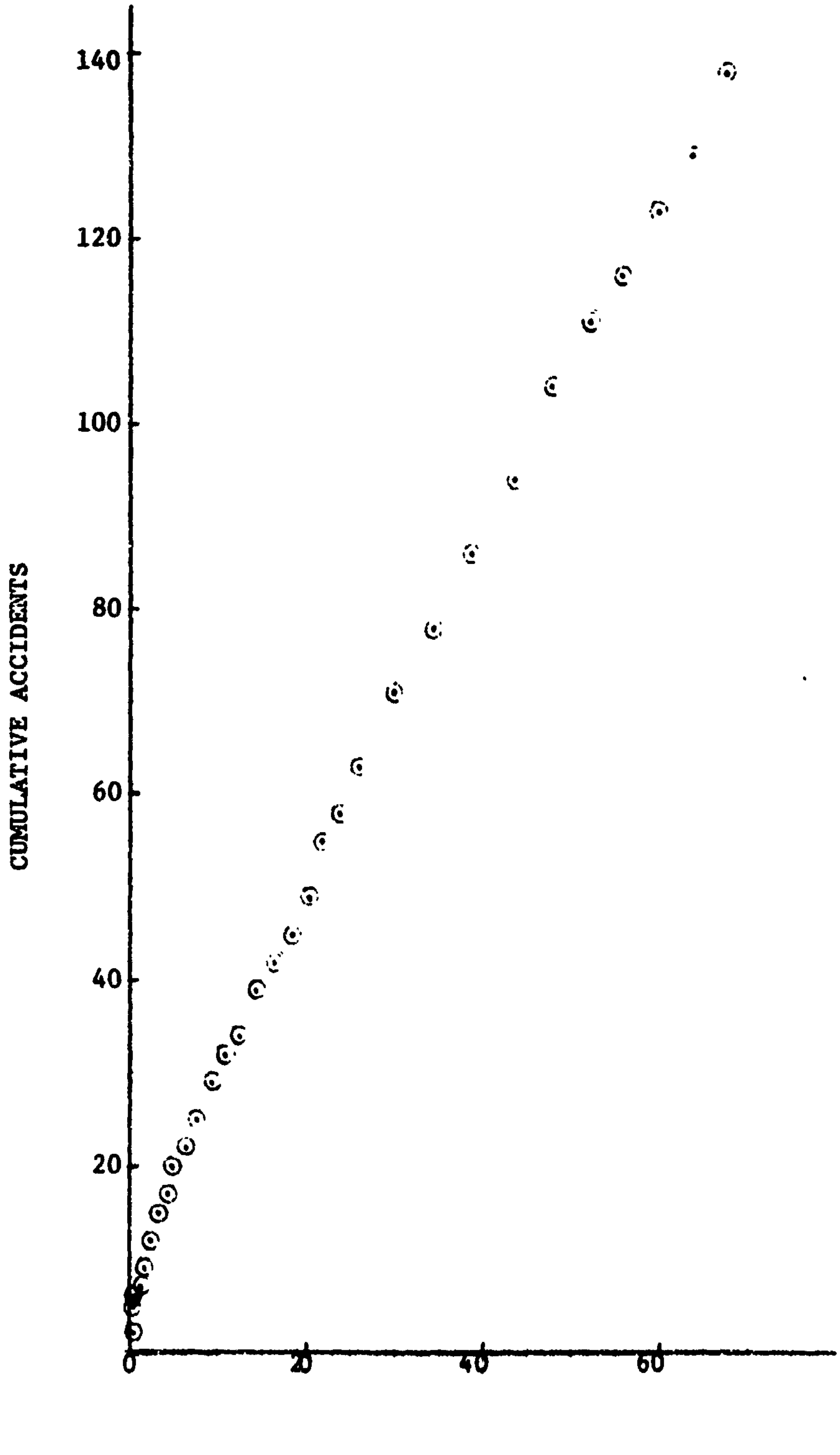
ALPHA = 1.59876156 GAMMA = 1.94672012 MU = 0.97252291

NUMBER OF ACCIDENTS	CUMULATIVE AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
10	3.0158	3.1137
20	6.4653	2.7453
30	10.3365	2.4665
40	14.6026	2.2555
50	19.2256	2.0958
60	24.1614	1.9749
70	29.3638	1.8835
80	34.7879	1.8142
90	40.3979	1.7618
100	46.1432	1.7222
110	52.0084	1.6922
120	57.9638	1.6695
130	63.9893	1.6523
140	70.0690	1.6393

FIGURE 9

CASE 2

ACTUAL AIRCRAFT ACCIDENT DATA



CUMULATIVE AIRCRAFT HOURS X 10^{-4}

FIGURE 10

CASE 2

ACTUAL CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE

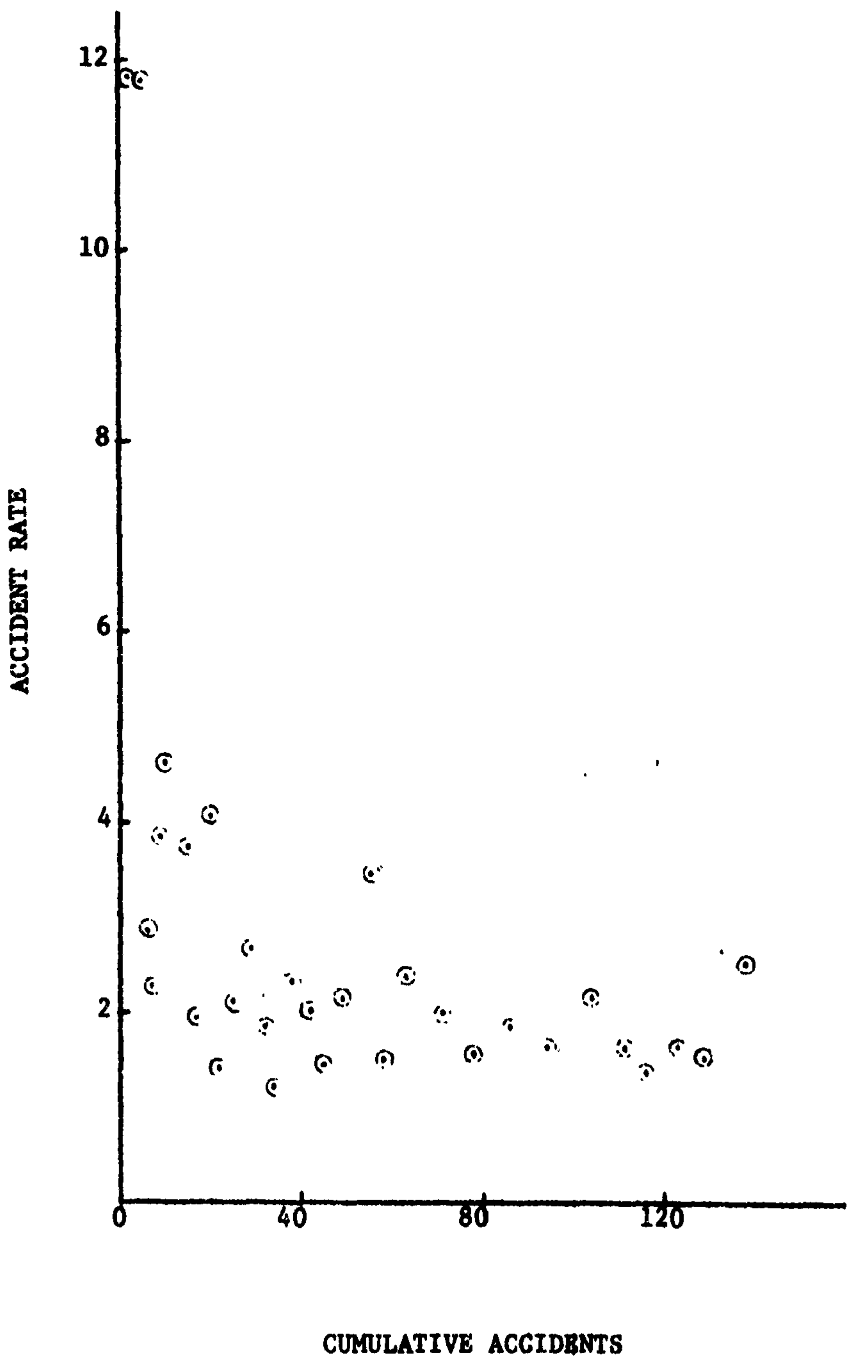


FIGURE 11

CASE 2

APPROXIMATE AIRCRAFT ACCIDENT CURVE

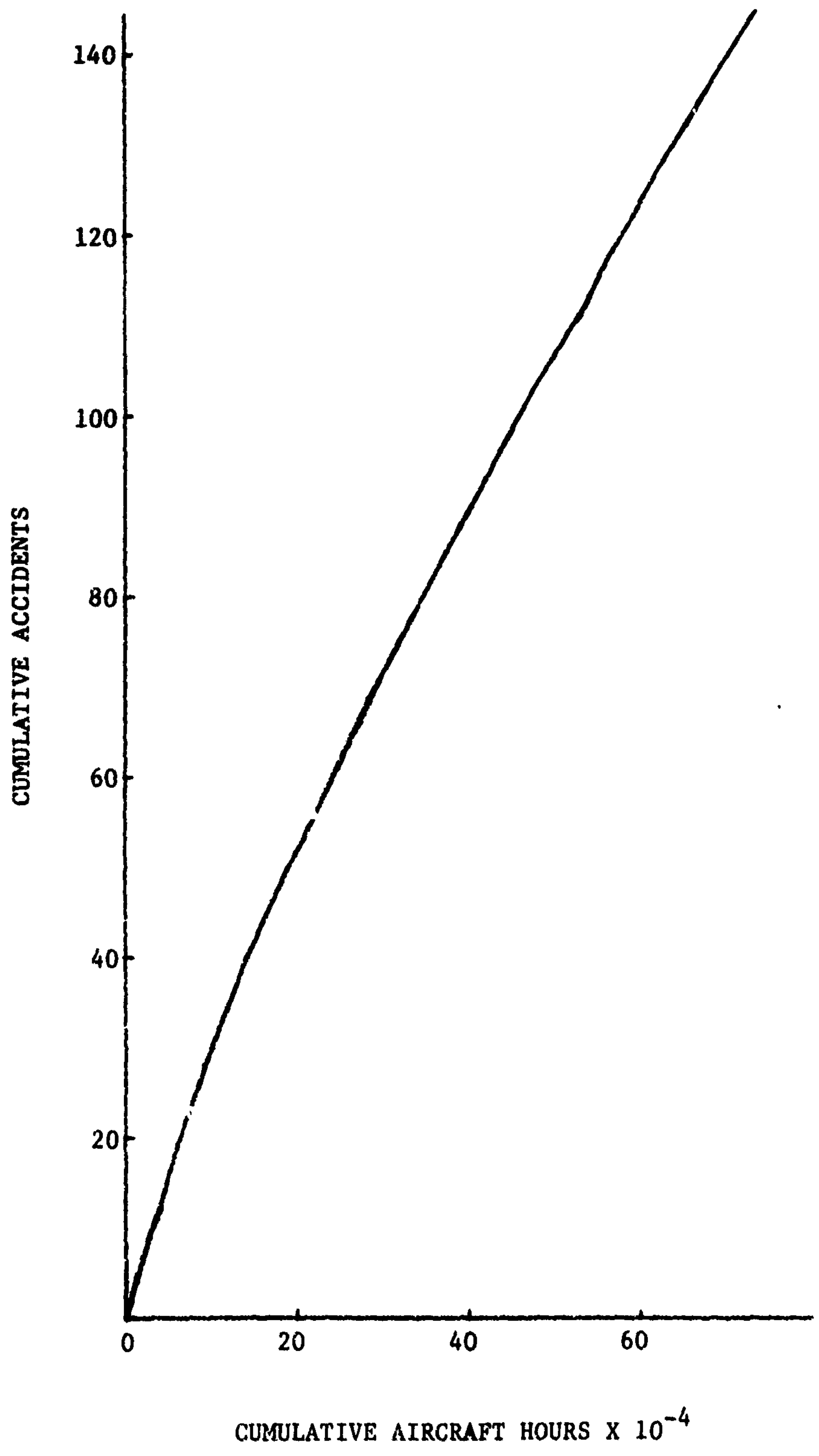
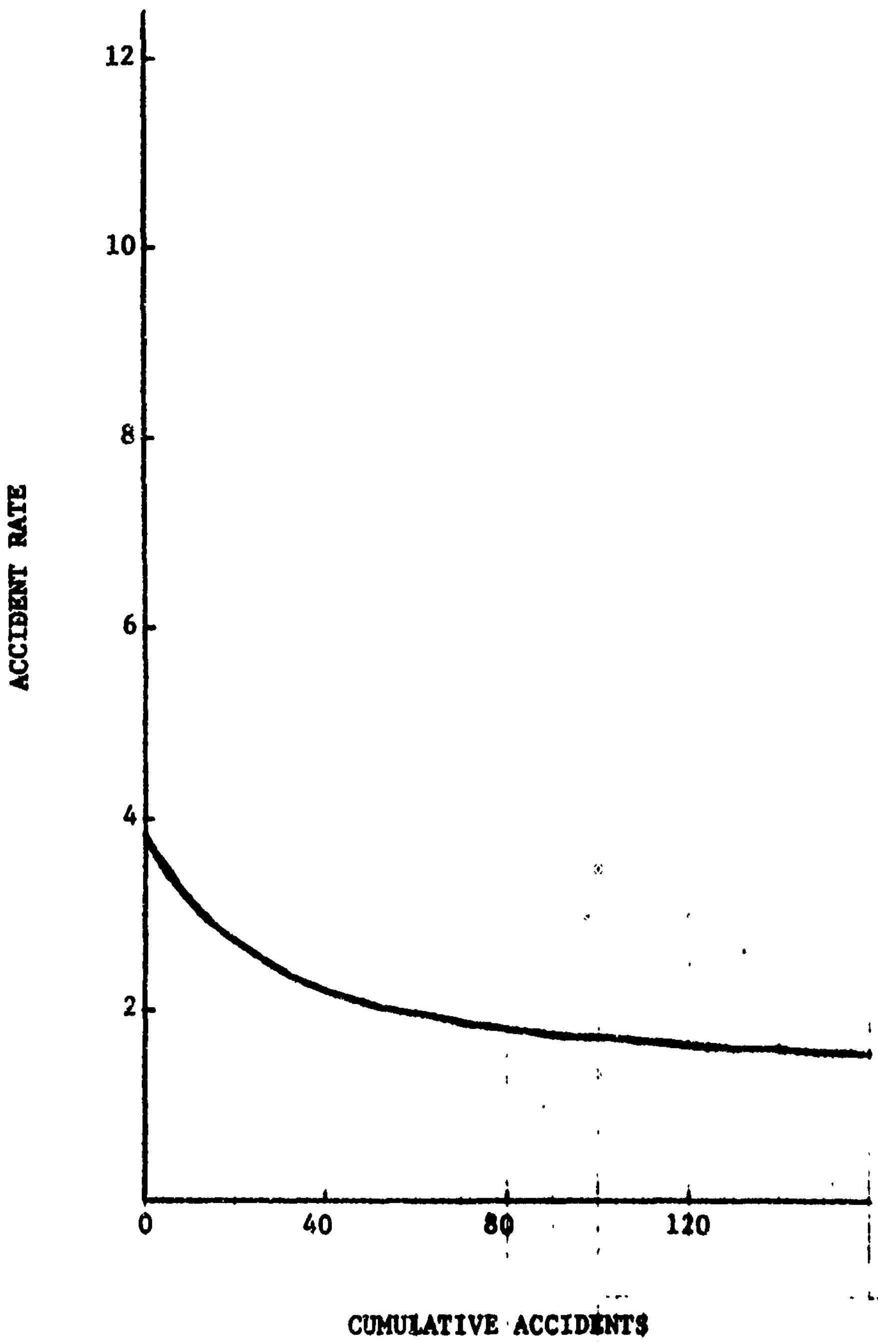


FIGURE 12

CASE 2

APPROXIMATE CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE



CASE 3

INPUT DATA

NUMBER OF ACCIDENTS	AIRCRAFT HOURS
1.	64.
0.	3.
2.	130.
7.	334.
4.	681.
4.	1827.
1.	973.
1.	719.
3.	1430.
2.	1512.
3.	1726.
4.	2510.
4.	3290.
4.	3221.
4.	2247.
4.	2768.
5.	3385.
5.	4197.
5.	3990.
7.	4509.
5.	5512.
13.	9168.
11.	8676.
12.	10285.
6.	12499.
15.	11315.
12.	12271.
13.	15088.
3.	6869.
7.	3792.
11.	7083.
0.	5482.

MODIFIED CUMULATIVE INPUT DATA

NUMBER OF ACCIDENTS	AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
1.	0.0064	156.2501
3.	0.0197	150.3760
10.	0.0531	209.5809
14.	0.1212	58.7372
18.	0.3039	21.8938
19.	0.4012	10.2775
20.	0.4731	13.9082
23.	0.6161	20.9790
25.	0.7073	13.2275
28.	0.9399	17.3812
32.	1.1909	15.9363
36.	1.5199	12.1581
40.	1.8420	12.4185
44.	2.0667	17.8015
48.	2.3435	14.4509
53.	2.6820	14.7710
58.	3.1017	11.9133
63.	3.5007	12.5313
70.	3.9516	15.5245
75.	4.5028	9.0711
88.	5.4196	14.1798
99.	6.2872	12.6787
111.	7.3157	11.6675
117.	8.5656	4.8004
132.	9.6971	13.2567
144.	10.9242	9.7792
157.	12.4330	8.6161
160.	13.1199	4.3674
167.	13.4991	18.4599
178.	14.2074	15.5301

INPUT INITIAL AND FINAL RATES AND INTERMEDIATE RATE AND CUMULATIVE ACCIDENTS USING FORMAT 4F5.1
M.0077 ENTER DATA.
42.0 10.0 12.5 60.0

SUCCESSIVE APPROXIMATIONS

ALPHA	GAMMA	MU	Q
10.00000000	32.00000000	0.95839936	0.59225416
9.89008331	33.21765137	0.95824713	0.57824373
9.86445332	32.56520184	0.95901471	0.57798862
9.86572552	32.60165405	0.95897353	0.57799006

OUTPUT DATA

ALPHA = 9.86445332 GAMMA = 32.56520184 MU = 0.95901471

NUMBER OF ACCIDENTS	CUMULATIVE AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
10	0.2721	32.2095
20	0.6347	24.5084
30	1.0990	19.5403
40	1.6685	16.2315
50	2.3381	14.0542
60	3.0954	12.6215
70	3.9243	11.6737
80	4.8082	11.0583
90	5.7325	10.6501
100	6.6856	10.3814
110	7.6586	10.2046
120	8.6451	10.0883
130	9.6407	10.0118
140	10.6425	9.9614
150	11.6483	9.9282
160	12.6509	9.9064
170	13.6672	9.8921
180	14.6786	9.8826

FIGURE 13

CASE 3

ACTUAL AIRCRAFT ACCIDENT DATA

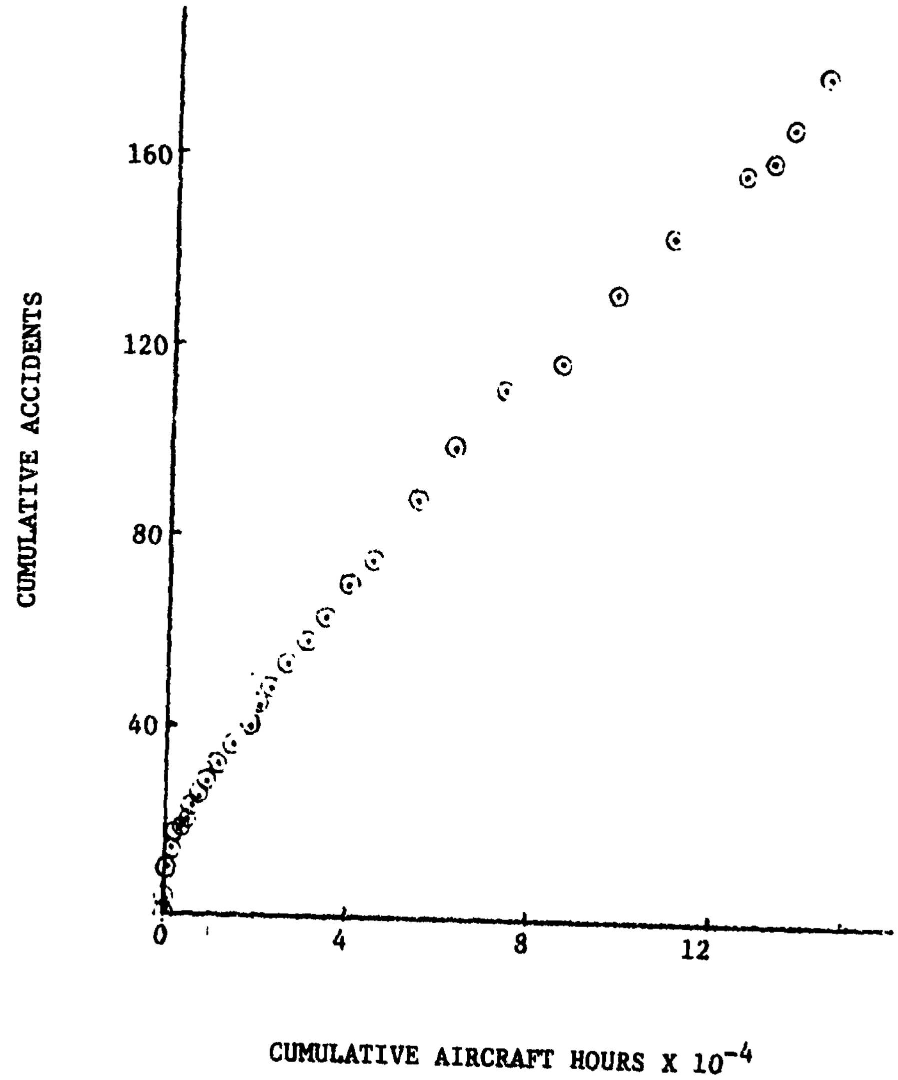


FIGURE 14

CASE 3

ACTUAL CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE

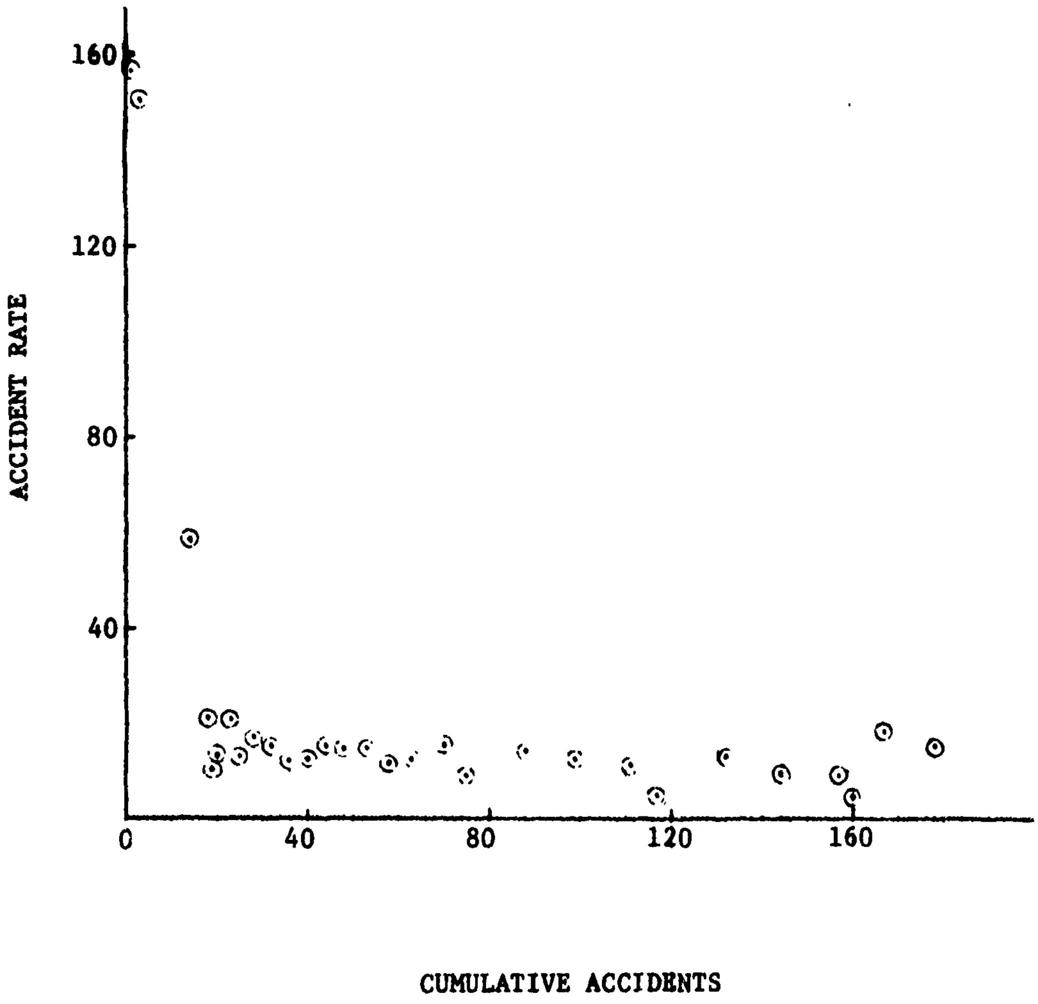


FIGURE 15

CASE 3

APPROXIMATE AIRCRAFT ACCIDENT CURVE

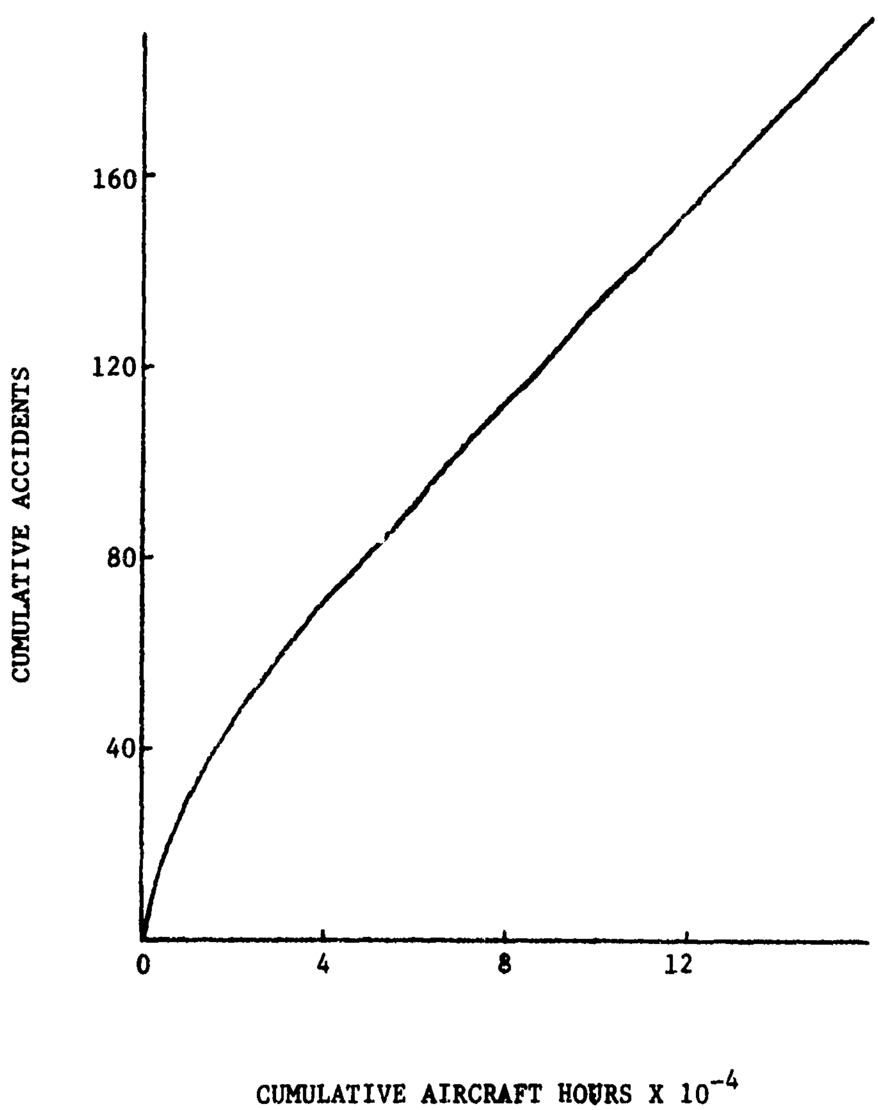
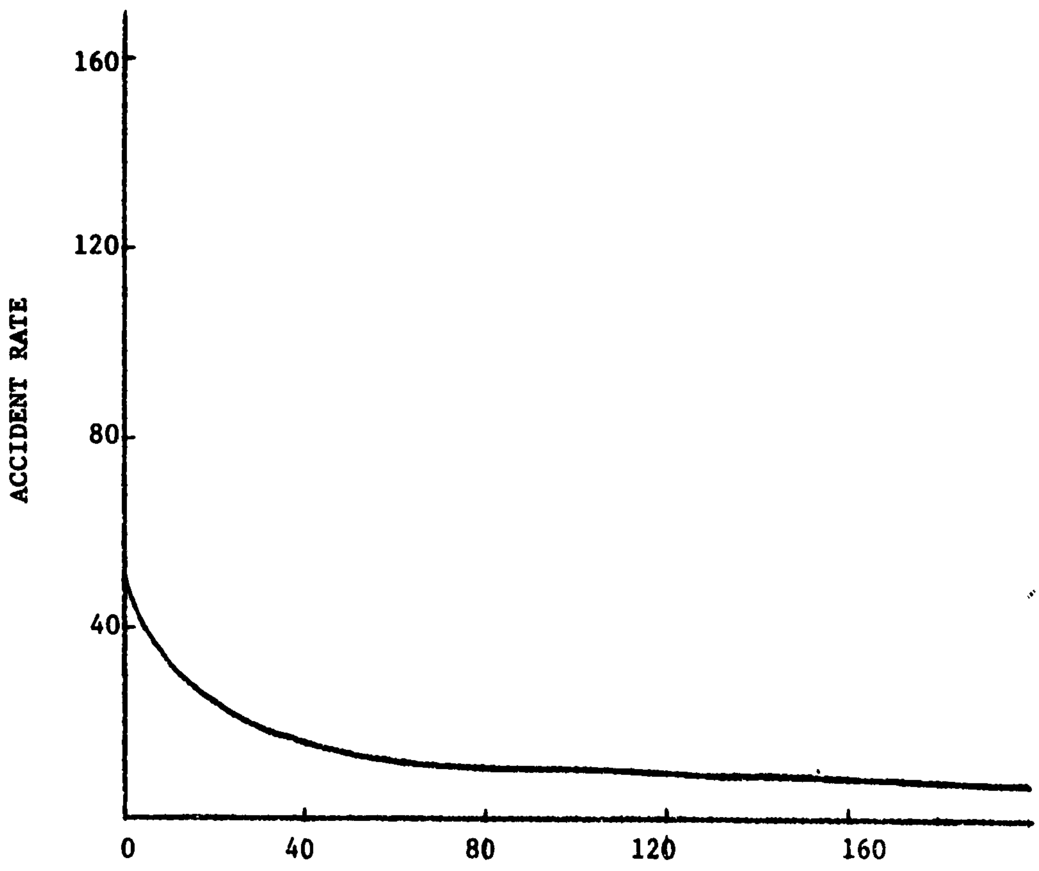


FIGURE 16

CASE 3

APPROXIMATE CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE



CUMULATIVE ACCIDENTS

CASE 4

NOT REPRODUCIBLE

INPUT DATA

NUMBER OF ACCIDENTS	AIRCRAFT HOURS
Ø.	64.
Ø.	3.
Ø.	13Ø.
2.	334.
1.	681.
1.	1827.
Ø.	973.
Ø.	719.
1.	143Ø.
1.	1512.
1.	1726.
2.	251Ø.
1.	329Ø.
1.	3221.
2.	2247.
1.	2768.
2.	3385.
1.	4197.
2.	399Ø.
3.	45Ø9.
2.	5512.
5.	9168.
5.	8676.
3.	1Ø285.
1.	12499.
4.	11315.
3.	12271.
6.	15Ø88.
2.	6869.
2.	3792.
5.	7Ø83.
Ø.	5482.

MODIFIED CUMULATIVE INPUT DATA

NUMBER OF ACCIDENTS	AIRCRAFT HOURS X 1Ø**(-4)	ACCIDENT RATE
2.	Ø.Ø531	37.6648
3.	Ø.1212	14.6843
4.	Ø.3Ø39	5.4735
5.	Ø.6161	3.2Ø31
6.	Ø.7673	6.6138
7.	Ø.9399	5.7937
9.	1.19Ø9	7.9681
1Ø.	1.5199	3.Ø395
11.	1.842Ø	3.1Ø46
13.	2.ØØ67	8.9ØØ8
14.	2.3435	3.6127
16.	2.682Ø	5.9Ø84
17.	3.1Ø17	2.3827
19.	3.5ØØ7	5.Ø125
22.	3.9516	6.6534
24.	4.5Ø28	3.6284
29.	5.4196	5.4538
34.	6.2872	5.763Ø
37.	7.3157	2.9169
38.	8.5656	Ø.8ØØ1
42.	9.6971	3.5351
45.	1Ø.9242	2.4448
51.	12.433Ø	3.9767
53.	13.1199	2.9116
55.	13.4991	5.2743
6Ø.	14.2Ø74	7.Ø592

INPUT INITIAL AND FINAL RATES AND INTERMEDIATE RATE AND CUMULATIVE
ACCIDENTS USING FORMAT 4F5.1
M.ØØ77 ENTER DATA.
8.Ø 3.Ø 4.Ø 3Ø.Ø

SUCCESSIVE APPROXIMATIONS

ALPHA	GAMMA	MU	Q
3.00000000	5.00000000	0.94776571	3.32456112
2.73181629	5.05465126	0.94653177	1.01308098
2.71273613	4.78335667	0.95164692	1.53015137
3.51560020	5.04265308	0.92134029	3.36095905

OUTPUT DATA

ALPHA = 2.71273613 GAMMA = 4.78335667 MU = 0.95164692

NUMBER OF ACCIDENTS	CUMULATIVE AIRCRAFT HOURS X 10**(-4)	ACCIDENT RATE
10	1.5306	5.7748
20	3.5124	4.5781
30	5.9293	3.8491
40	8.7206	3.4050
50	11.8037	3.1345
60	15.0971	2.9697
70	18.5333	2.8693

FIGURE 17

CASE 4

ACTUAL AIRCRAFT ACCIDENT DATA

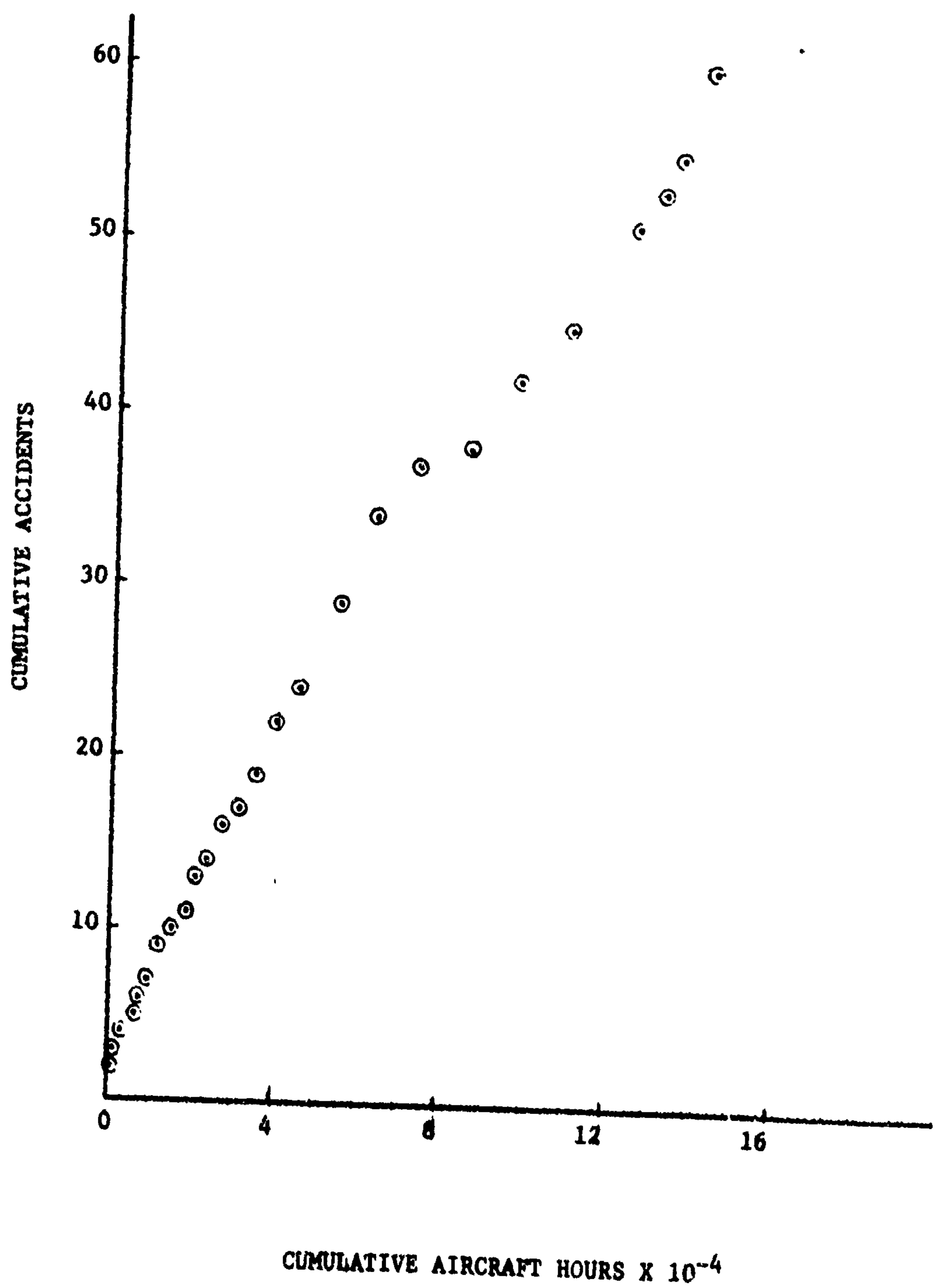


FIGURE 18

CASE 4

ACTUAL CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE

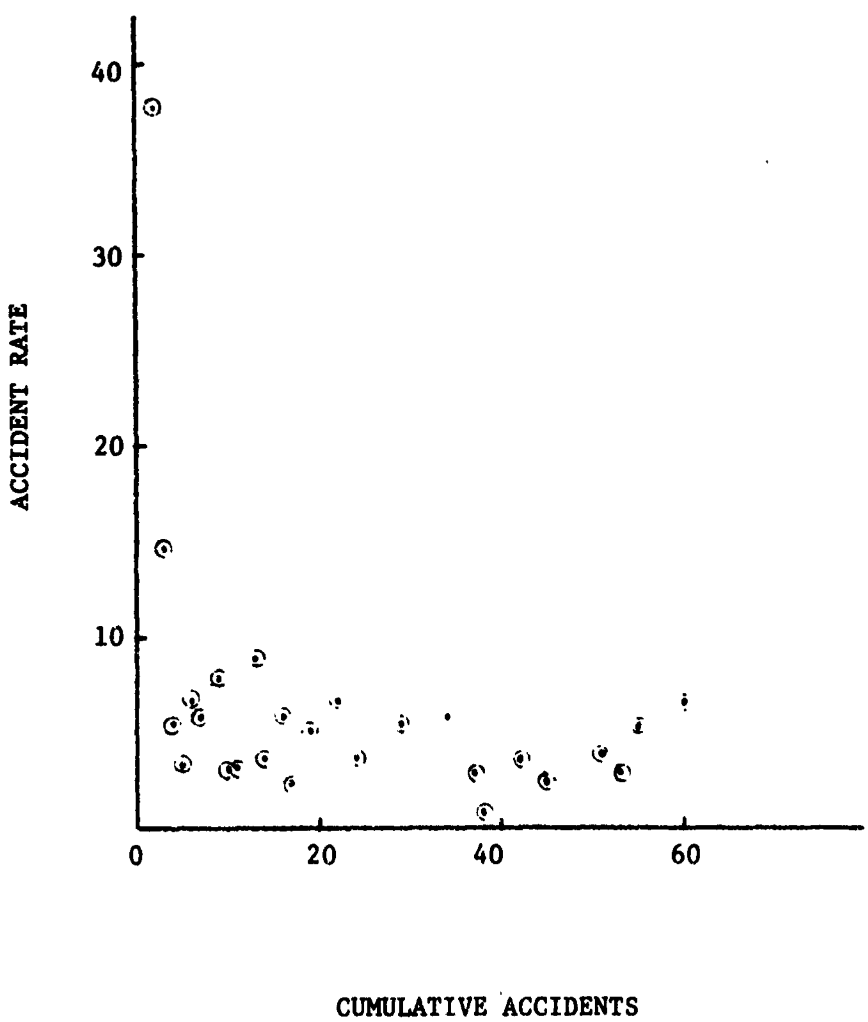


FIGURE 19

CASE 4

APPROXIMATE AIRCRAFT ACCIDENT CURVE

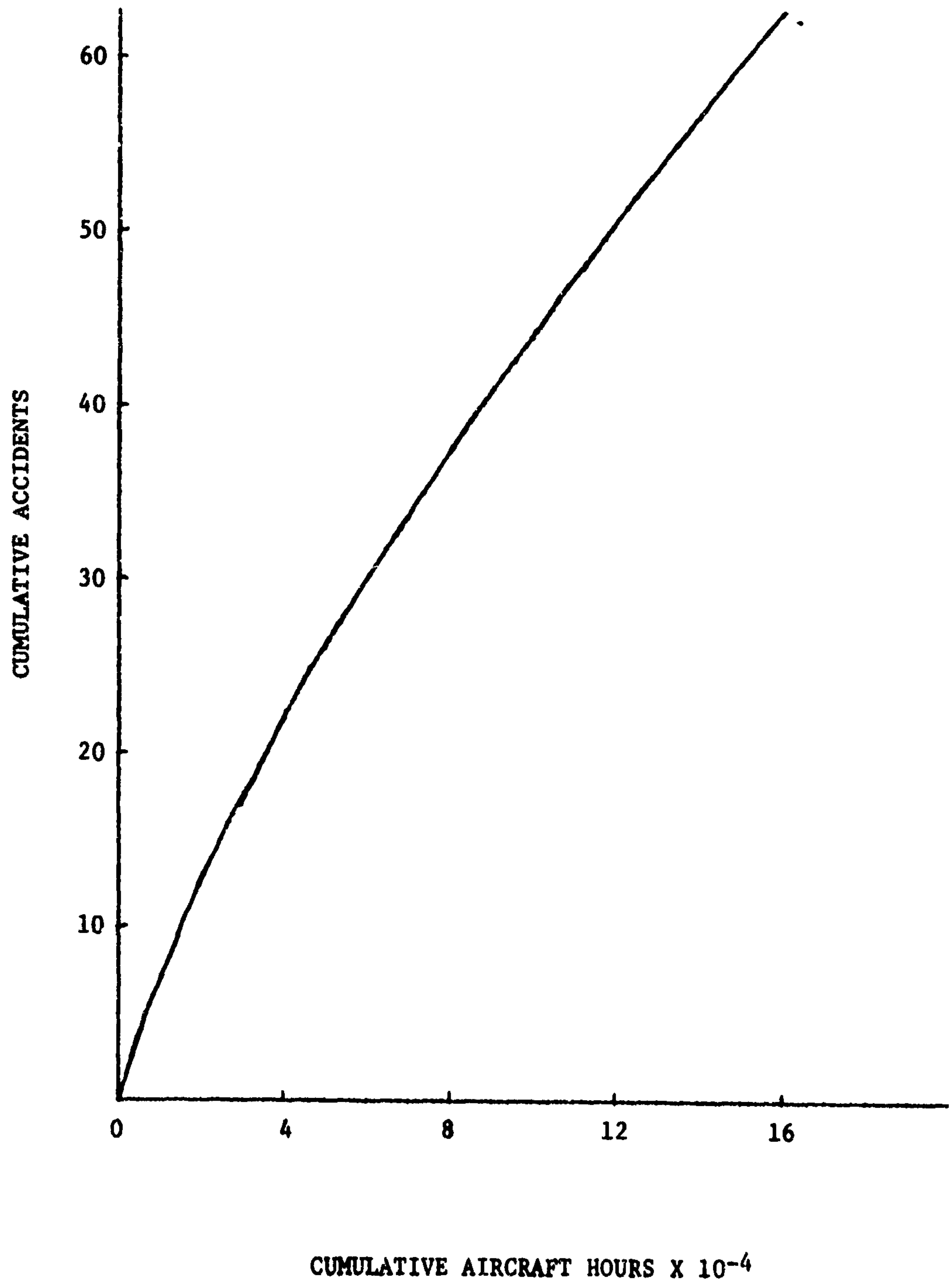
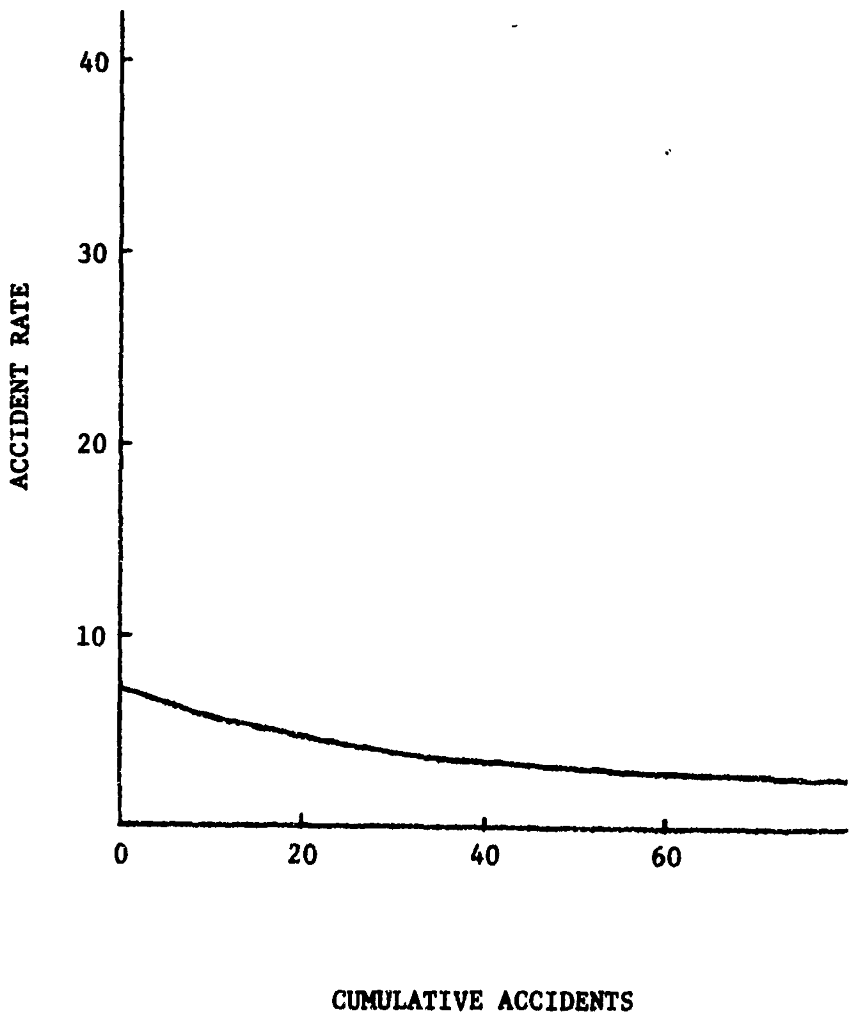


FIGURE 20

CASE 4

APPROXIMATE CUMULATIVE ACCIDENTS VERSUS ACCIDENT RATE



SUMMARY

This report has presented a method of predicting aircraft accidents using the pure birth process and relating accident rates directly to the total number of past accidents.

Other methods used to predict aircraft attrition were also included in order to demonstrate weaknesses and strengths in these various methods and in the pure birth method. Then depending on particular characteristics of a case, the most accurate prediction method can be employed.

APPENDIX. PARTIAL DERIVATIVES OF Q

$$Q = (\alpha,\ \gamma,\ \mu)$$

$$= \tfrac{1}{2} \sum_{k=1}^{m} \{ \sum_{j=0}^{n_k-1} \frac{1}{\lambda_j} - t_k \}^2 = \tfrac{1}{2} \sum_{k=1}^{m} F_k{}^2$$

where $\lambda_j = \alpha + \gamma\mu^j, \qquad E_k = \sum_{j=0}^{n_k-1} \frac{1}{\lambda_j} - t_k.$

$$\frac{\partial E_k}{\partial \alpha} = - \sum \frac{1}{\lambda_j{}^2}$$

$$\frac{\partial E_k}{\partial \gamma} = - \sum \frac{\mu^j}{\lambda_j{}^2}$$

$$\frac{\partial E_k}{\partial \mu} = - \frac{\gamma}{\mu} \Sigma \frac{j\mu^j}{\lambda_j{}^2}$$

$$\frac{\partial^2 E_k}{\partial \alpha^2} = 2\Sigma \frac{1}{\lambda_j{}^3}$$

$$\frac{\partial^2 E_k}{\partial \gamma^2} = 2\Sigma \frac{(\mu^j)^2}{\lambda_j{}^3}$$

$$\frac{\partial^2 E_k}{\partial \mu^2} = \frac{2\gamma^2}{\mu^2} \Sigma \frac{j^2(\mu^j)^2}{\lambda_j{}^3} - \frac{\gamma}{\mu^2} \Sigma \frac{j(j-1)\mu^j}{\lambda_j{}^2}.$$

$$\frac{\partial^2 E_k}{\partial \alpha \partial \gamma} = 2\Sigma \frac{\mu^j}{\lambda_j{}^3}$$

$$\frac{\partial^2 E_k}{\partial \alpha \partial \mu} = \frac{2\gamma}{\mu} \Sigma \frac{j\,\mu^j}{\lambda_j{}^3}$$

$$\frac{\partial^2 E_k}{\partial \gamma \partial \mu} = \frac{2\gamma}{\mu} \Sigma \frac{j(\mu^j)^2}{\lambda_j{}^3} - \frac{1}{\mu} \Sigma \frac{j\mu^j}{\lambda_j{}^2}.$$

All of the summations indicated above run from $j=0$ to $j=n_k-1$. The summations below run from $k=1$ to $k=m$.

$$\frac{\partial Q}{\partial \alpha} = \Sigma\, E_k \;\frac{\partial E_k}{\partial \alpha}$$

$$\frac{\partial Q}{\partial \gamma} = \Sigma\, E_k \;\frac{\partial E_k}{\partial \gamma}$$

$$\frac{\partial Q}{\partial \mu} = \Sigma\, E_k \;\frac{\partial E_k}{\partial \mu}$$

$$\frac{\partial^2 Q}{\partial \alpha^2} = \Sigma[\,(\frac{\partial E_k}{\partial \alpha})^2 + E_k \frac{\partial^2 E_k}{\partial \alpha^2}\;]$$

$$\frac{\partial^2 Q}{\partial \gamma^2} = \Sigma[\,(\frac{\partial E_k}{\partial \gamma})^2 + E_k \frac{\partial^2 E_k}{\partial \gamma^2}\;]$$

$$\frac{\partial^2 Q}{\partial \mu^2} = \Sigma[\,(\frac{\partial E_k}{\partial \mu})^2 + E_k \frac{\partial^2 E_k}{\partial \mu^2}\;]$$

$$\frac{\partial^2 Q}{\partial \alpha \partial \gamma} = \frac{\partial^2 Q}{\partial \gamma \partial \alpha} = \Sigma[\,(\frac{\partial E_k}{\partial \alpha})(\frac{\partial E_k}{\partial \gamma}) + E_k \frac{\partial^2 E_k}{\partial \alpha \partial \gamma}\,]$$

$$\frac{\partial^2 Q}{\partial \alpha \partial \mu} = \frac{\partial^2 Q}{\partial \mu \partial \alpha} = \Sigma[\,(\frac{\partial E_k}{\partial \alpha})(\frac{\partial E_k}{\partial \mu}) + E_k \frac{\partial^2 E_k}{\partial \alpha \partial \mu}\,]$$

$$\frac{\partial^2 Q}{\partial \gamma \partial \mu} = \frac{\partial^2 Q}{\partial \mu \partial \gamma} = \Sigma[\,(\frac{\partial E_k}{\partial \gamma})(\frac{\partial E_k}{\partial \mu}) + E_k \frac{\partial^2 E_k}{\partial \gamma \partial \mu}\,]$$

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13. ABSTRACT

Non-combat aircraft accident statistics indicate that a direct relationship exists between the number of accidents and accumulated flight hours or similarly between the accident rate and accumulated flight hours for each model of military airplane. This paper investigates the feasibility of relating accident rates directly to the total number of past accidents.

Based on the pure birth process a method for predicting aircraft accidents is presented. Application of this procedure to various test cases shows interesting and useful results. One definite conclusion that can be drawn is that with two or more years of flight and accident data, future aircraft accident rates can be predicted with fairly reliable accuracy.

Future studies based on these same procedures will delve into further relationships that may exist between aircraft characteristics and other relevant accident factors.

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
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